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Human Capital, Physical Capital, Technological Development, and Economic Growth in Imo State, Nigeria: A Mathematical Modeling Perspective

Abstract

This study develops and analyses a deterministic mathematical model for investigating the interaction among human capital, physical capital, technological development, and economic growth in Imo State, Nigeria. The model is formulated as a system of three ordinary differential equations describing the evolution of human capital, physical capital, and technological development, while aggregate economic output is represented by a Cobb–Douglas production function. The formulation incorporates the reciprocal influence between human capital and technological development, alongside the accumulation of physical capital through investment from aggregate economic output. The qualitative properties of the model are examined by establishing the existence and uniqueness of solutions, positivity, and boundedness of the state variables. Furthermore, the equilibrium analysis yields a unique positive equilibrium under appropriate parameter restrictions. Local stability analysis, based on the Jacobian matrix and eigenvalue criteria, establishes that the positive equilibrium is locally asymptotically stable when the product of the human-capital and technological depreciation rates exceeds the product of their interaction parameters and the physical-capital output elasticity lies between zero and unity. These results provide mathematical conditions under which the model generates positive, bounded economic trajectories and approaches a sustainable long-run equilibrium. The proposed framework highlights the interconnected roles of human-capital development, physical-capital accumulation, and technological advancement in shaping economic growth. It also provides a theoretical foundation for subsequent numerical simulations, parameter estimation, and sensitivity analysis aimed at understanding the dynamics of economic development in Imo State, Nigeria.

Keywords: Deterministic mathematical model; Economic growth; Human capital; Physical capital; Technological development; Cobb–Douglas production function; Imo State, Nigeria.

1 Introduction

Economic growth remains one of the central concerns of development economics and public policy because of its implications for income generation, employment creation, poverty reduction, and improvements in living standards. The ability of an economy to sustain growth over time depends on several interrelated factors, including the accumulation of physical capital, the development of human resources, technological advancement, and the efficient utilisation of available productive resources. Understanding how these factors interact is therefore important for formulating appropriate development strategies, particularly in developing economies where structural constraints may limit productive capacity. The theoretical foundations of modern economic growth analysis can be traced to the neoclassical growth framework developed by Solow [2] and Swan [3], which established the importance of capital accumulation, labour, and technological progress in explaining long-run economic performance. Their contributions provided a foundation for subsequent investigations into the determinants of economic growth and the mechanisms through which productive resources influence aggregate output.

Physical capital accumulation has traditionally been recognised as an important determinant of economic growth. Investments in machinery, industrial facilities, transportation networks, energy systems,

and other productive assets expand the capacity of an economy to produce goods and services. The relationship between productive inputs and aggregate output is commonly represented using the Cobb–Douglas production function introduced by Cobb and Douglas [1]. This production framework provides a convenient mathematical representation of the contributions of different productive factors to aggregate output and permits the estimation of their respective output elasticities. Within the neoclassical growth framework, capital accumulation contributes to economic expansion, although diminishing marginal returns to individual productive factors imply that sustained long-term growth also requires improvements in productivity and technological efficiency [2, 3]. The importance of investment in productive assets has also received considerable attention in empirical studies. For example, Aschauer [11] documented the importance of productive public capital in growth, while Barro [10] included productive government services into an endogenous growth framework. While this paper does not attempt to separate public and private capital, these contributions reinforce the more general importance of capital in growth. Beyond physical capital, human capital is another essential determinant of development. Human capital can be dened as the knowledge, education, skills, experience and productive capacities embodied in a person. Expenditure on education, health, training and professional development increase productivity by making workers more productive and increasing their chances of making contributions to production. Schultz [4] demonstrated the effect of investment in human capital on economic productivity, and Becker [5] provided a more formal economic framework of education, training and investment in human capital. Subsequently, many growth models incorporated human capital as a factor in the production function. Lucas [6] showed that human capital accumulation can generate long-run growth due to increasing returns in human capital investment and the spillover effects on the economy, while Mankiw, Romer and Weil [9] incorporated human capital accumulation into the neoclassical growth model alongside physical capital and showed that the augmented neoclassical model gives a good account of international differences in income levels. These papers justify the theoretical importance of including human capital as an explicit productive factor in economic growth models. Technological development is important since it determines the efficiency with which other productive factors are combined.

Technological development can increase efficiency in production, improve the availability of information, encourage innovation, reduce costs of production and provide new avenues for economic activity. In the neoclassical growth model, technological development is an important source of long-run growth in output per worker [2]. Endogenous growth theory provided an explanation for technological development as an internal result of the investment and innovation decision. Romer [7] showed that knowledge can provide for long-run economic growth due to increasing returns and positive externalities, and Romer [8] formalized technological change as a result of research and development investment. Aghion and Howitt [12] explained technological development through the idea of creative destruction, where innovation replaces existing technologies and results in overall productivity gains. These theoretical developments demonstrate that technological development should not necessarily be treated as an exogenous factor, but rather as related to development of human resources and accumulation of productive assets. The relationship between human capital and technological development is especially important in developing countries; a well-educated and trained work force is more likely to make use of technological developments, and advances in technology and access to information is likely to improve education outcomes and training possibilities.

Benhabib and Spiegel [13] studied the relationship between human capital and economic development, and Romer [8] highlighted the importance of human capital in technological development. These results suggest that human capital and technological development should not be treated as two separate exogenous productive factors, but rather as related. In this paper, a two-way relationship is assumed: technological development can increase human capital formation, while human capital can increase technological development. In Nigeria, the relationship between productive resources and economic growth has been widely studied empirically. Despite the country's vast human and natural resources, insufcient infrastructure, poor industrial development, insufcient investment and lack of human capital are still hindrances to growth. Empirical studies have investigated the effects of education, technological development, capital formation and other macroeconomic variables on economic growth. Sulaiman et al. [14] studied the relationship between human capital, technology and economic growth in Nigeria, and found that secondary and tertiary education enrollment have positive signicant effects on economic growth.

Keji [15] also studied the human capital and economic growth relationship in Nigeria, and found that there is a signicant long run relationship between human capital and economic growth. These results support consideration of human capital formation as a factor of economic growth. The role of physical capital formation and infrastructure on economic growth in Nigeria has also been studied empirically; Onwioduokit and Otolorin [16] studied capital formation and economic growth in Nigeria using annual data from 1981 to 2018. Human capital formation was found to have a positive effect on economic growth, but the effect of gross xed capital formation was negative and signicant in their model specification. Similarly, Opadeji et al. [17] studied gross capital formation, infrastructure and economic

development in Nigeria, and found that infrastructure has a positive and signicant effect on economic development, but gross capital formation does not in their estimated model. Eniekezimene, Wodu and Anda-Owei [18] studied human capital formation and economic growth in Nigeria using data from 1981 to 2021. The varied results of these studies show that the effect of capital formation, both physical and human, on economic growth is likely to depend on which variables are included, how they are measured, and which econometric method is used. Most importantly, the results point to the importance of a proper framework in which the relationships between productive factors are properly specied.

Though national level studies on the factors affecting economic growth in Nigeria have been carried out, the development level of states may be very different due to differences in endowments of productive resources such as infrastructure, education and training opportunities, industrialization and technology. Thus, lessons drawn from aggregate national-level analysis may not be accurate on a state level. Recently, more state-level economic accounts have been published, highlighting the huge differences in the level of economic activity between states in Nigeria [19]. In the case of Imo State, state economic accounts identify construction, trade, agriculture, services and other economic activities as key sectors. To realize its development potential, productive assets and interactions that affect state-level output must be considered. A mathematical framework that explicitly models the interactions between human capital, physical capital and technological development will be very useful in this regard.

Mathematical modeling of economic relationships has an advantage in that the interactions between productive factors can be represented as differential equations, and the dynamic nature of economic growth can be considered. The investigation of the effect of changes in one variable on the long-run levels of other variables is possible. In this paper, deterministic ordinary differential equations are used to model the dynamics of physical capital, human capital and technological development. The Cobb-Douglas production function is used to represent economic output as a function of the three productive factors. The positivity and boundedness of solutions, as well as the existence and stability of equilibrium points, can be established. These mathematical properties are important because they reveal the parameter conditions required for the model to have economically feasible solutions and to approach equilibrium.

Inspired by these arguments, in this paper, a deterministic mathematical model is proposed to describe the dynamics of human capital, physical capital, technological development and economic growth in Imo State, Nigeria. The model comprises three ordinary differential equations modeling the dynamics of human capital, physical capital and technological development, and a Cobb-Douglas production function for economic output. The effect of technological development on human capital formation is modeled, as well as the effect of human capital on technological development, and the accumulation of physical capital through investment from economic output. Thus, the two-way relationship between human capital and technology is captured, as is the contribution of physical capital to economic output. The main contribution of this paper is the development and qualitative analysis of a compact deterministic growth model in a subnational economic context. The existence and uniqueness of solutions to the model is investigated, and the conditions for positivity and boundedness of solutions are established. The positive equilibrium point of the system is determined and its existence proven.

The conditions required for local asymptotic stability of the positive equilibrium point are determined. Explicit conditions on the system parameters for the existence of a unique positive equilibrium point, and for stability of the equilibrium point, are provided. These results can be used to guide the design of experiments to simulate the dynamics of the system under different values of the parameters. The conditions for boundedness and for existence and uniqueness of solutions presented here can also serve as a foundation for further quantitative analysis of the economic growth model, and for numerical experiments and sensitivity analysis of the model's behavior under different investment, depreciation, human capital and technological development parameters. By focusing on the interactions between the three productive factors, the model provides a foundation for mathematical modeling of economic growth at the subnational level.

2 Model Formulation

This study develops a deterministic mathematical model to examine the interaction among human capital, physical capital, technological development, and economic growth in Imo State, Nigeria. The model describes the evolution of the major productive factors through a system of ordinary differential equations, while aggregate economic output is represented by a Cobb–Douglas production function.

The formulation is based on the premise that economic growth depends on the accumulation of physical capital, the development of human resources, and technological advancement. These productive factors interact to determine the overall economic performance of the state.

The proposed model is given by

$$\begin{aligned} \frac{dH}{dt} &= a_H + \alpha_H T - \mu_H H, \\ \frac{dK}{dt} &= sY - \delta K, \\ \frac{dT}{dt} &= a_T + \alpha_T H - \mu_T T, \\ Y &= AK^{\gamma} H^{\beta} T^{\theta}. \end{aligned} \tag{1}$$

The initial conditions are specified as

$$H(0) = H_0 > 0, \qquad K(0) = K_0 > 0, \qquad T(0) = T_0 > 0. \tag{2}$$

Here, $t \geq 0$ denotes time, and the initial values represent the levels of human capital, physical capital, and technological development at the beginning of the study period.

2.1 Description of State Variables

The model consists of three dynamic state variables and one algebraic output variable, described as follows:

- $H(t)$ denotes human capital, representing the knowledge, education, skills, and productive capabilities of the workforce in Imo State.
- $K(t)$ denotes physical capital stock, representing productive assets such as machinery, industrial facilities, infrastructure, transportation networks, and other capital resources.
- $T(t)$ represents technological development, including technological adoption, digital advancement, internet accessibility, and innovation within the state's economy.
- $Y(t)$ denotes aggregate economic output, measured by the real gross domestic product (GDP) of Imo State.

2.2 Description of the Model Equations

The proposed model consists of three differential equations describing the evolution of human capital, physical capital, and technological development, together with a production function determining aggregate economic output.

2.2.1 Human Capital Dynamics

The evolution of human capital is governed by

$$\frac{dH}{dt} = a_H + \alpha_H T - \mu_H H. \tag{3}$$

This equation represents the evolution and depreciation of human capital in Imo State. The parameter a_H denotes the baseline rate of human capital formation via educational activities, training, professional development, and other human resource formation processes.

The term $\alpha_H T$ represents the contribution of technological progress to human capital formation. Technological advancement improves information access, availability of digital learning resources, educational opportunities, and other forms of skill acquisition. The parameter α_H is the technological progress coefficient.

Finally, the term $\mu_H H$ represents depreciation of human capital via technological obsolescence and other sources of decreasing human capital. Thus, the parameter μ_H is the human capital depreciation rate.

2.2.2 Physical Capital Dynamics

The accumulation of physical capital is described by

$$\frac{dK}{dt} = sY - \delta K. \tag{4}$$

This equation represents the accumulation of physical capital through investment and depreciation. The term sY denotes the proportion of aggregate economic output allocated to physical capital investment, where s is the investment rate. Such investment may include the acquisition of machinery, expansion of industrial facilities, construction of infrastructure, and development of other productive assets.

The parameter δ represents the depreciation rate of physical capital due to wear and tear, deterioration, and technological obsolescence. Consequently, physical capital increases when investment exceeds depreciation and declines when depreciation exceeds new investment.

2.2.3 Technological Development Dynamics

The evolution of technological development is governed by

$$\frac{dT}{dt} = a_T + \alpha_T H - \mu_T T. \tag{5}$$

This equation describes the development of technology through baseline technological advancement and human-capital development. The parameter a_T represents the baseline contribution to technological development arising from existing innovation, research activities, and technology-adoption initiatives.

The term $\alpha_T H$ captures the contribution of human capital to technological development. An educated and skilled workforce is better positioned to adopt, adapt, and utilise modern technologies. The parameter α_T measures the strength of this contribution.

The term $\mu_T T$ represents the loss of technological capacity through technological obsolescence and the gradual replacement of existing technologies. The parameter μ_T is therefore the technological obsolescence rate.

2.2.4 Aggregate Economic Output

Aggregate economic output is represented by the Cobb–Douglas production function

$$Y = AK^{\gamma} H^{\beta} T^{\theta}. \tag{6}$$

The parameter A is total factor productivity, which encompasses the effectiveness with which the productive factors are combined. The parameters γ, β, and θ denote the output elasticities of physical capital, human capital, and technological development respectively.

The production function implies that economic output is a function of the three productive factors. If one productive factor increases and the others are held constant, then economic output increases if the output elasticity of the increased factor is positive.

Under the constant-returns-to-scale assumption, the production elasticities satisfy

$$\gamma + \beta + \theta = 1, \qquad \gamma, \beta, \theta \geq 0. \tag{7}$$

This condition implies that a proportional increase in all three productive factors produces an equal proportional increase in aggregate economic output.

2.3 Description of Model Parameters

The parameters appearing in system (1) and their economic interpretations are summarised in Table 1.

2.4 Model Assumptions

The formulation of the model is based on the following assumptions:

1. Human capital increases through baseline human-resource development and technological advancement but decreases through depreciation and skill obsolescence.

2. Physical capital accumulates through investment, represented by a fixed proportion of aggregate economic output, and decreases through depreciation.

3. Technological development increases through baseline technological advancement and human-capital development but decreases through technological obsolescence.

4. Aggregate economic output depends on the combined contributions of physical capital, human capital, and technological development.

Table 1: Description of model parameters

Parameter	Description
a_H	Baseline contribution to human capital formation
α_H	Influence of technological development on human capital
μ_H	Human-capital depreciation rate
s	Proportion of output allocated to physical capital investment
δ	Depreciation rate of physical capital
a_T	Baseline contribution to technological development
α_T	Contribution of human capital to technological development
μ_T	Technological obsolescence rate
A	Total factor productivity
γ	Output elasticity of physical capital
β	Output elasticity of human capital
θ	Output elasticity of technological development

5. All state variables and model parameters are nonnegative, while the depreciation parameters μ_H, δ, and μ_T are strictly positive.

6. The investment rate satisfies $0 \leq s \leq 1$.

7. The production elasticities are nonnegative and satisfy the constant-returns-to-scale condition $\gamma + \beta + \theta = 1$.

8. The model is deterministic, and random economic shocks and other stochastic disturbances are not explicitly incorporated.

9. The model parameters are assumed constant throughout the period of analysis.

2.5 Economic Interpretation and Relevance to Imo State

This model provides an analytical tool for examining the linkages between human-capital formation, physical-capital accumulation, and technological progress, and the economic growth in Imo State.

The model makes explicit the complementarities between human capital and technological progress; improvements in technology may increase accessibility to education, information and training, while improvements in human capital increase the ability of the economy to absorb modern technologies.

Physical capital is central to the accumulation of productive resources, since a fraction of aggregate economic output is directed to physical investment, so that increases in output may lead to increases in physical capital; this gives rise to an indirect effect of increases in human capital and technology on investment, and thus economic growth.

On the whole, the model is a fairly simple framework for the study of the interactions between human capital, physical capital, technological change and economic growth in Imo State, Nigeria.

3 Qualitative Analysis of the Model

This section investigates the qualitative properties of the proposed deterministic economic growth model. In particular, we establish the positivity and boundedness of the model solutions, determine the equilibrium point, and examine the local asymptotic stability of the equilibrium. These analytical properties are important because they provide mathematical assurance that the model generates economically meaningful trajectories and that its long-term behaviour can be properly understood.

For convenience, the governing system is rewritten as (1)

Throughout the analysis, we assume that all model parameters are nonnegative, with

$$a_H, a_T, \alpha_H, \alpha_T, \mu_H, \mu_T, s, A, \delta > 0,$$

and that the production elasticities satisfy

$$0 < \gamma < 1, \qquad \beta, \theta > 0, \qquad \gamma + \beta + \theta = 1.$$

The initial conditions are also assumed to be strictly positive.

3.1 Existence and Uniqueness of Solutions

We first establish the existence and uniqueness of solutions to system (1). Define the state vector

$$X(t) = \begin{pmatrix} H(t) \\ K(t) \\ T(t) \end{pmatrix}.$$

The system can be expressed in the compact form

$$\frac{dX}{dt} = F(X), \tag{8}$$

where

$$F(X) = \begin{pmatrix} a_H + \alpha_H T - \mu_H H \\ sAK^{\gamma}H^{\beta}T^{\theta} - \delta K \\ a_T + \alpha_T H - \mu_T T \end{pmatrix}.$$

For strictly positive values of H, K, and T, the function $F(X)$ is continuously differentiable and consequently locally Lipschitz continuous. Therefore, by the standard existence and uniqueness theorem for ordinary differential equations, for every strictly positive initial condition

$$H(0) = H_0 > 0, \qquad K(0) = K_0 > 0, \qquad T(0) = T_0 > 0,$$

there exists a unique local solution of system (1).

The positivity and boundedness results established in the subsequent subsections ensure that this solution remains in a bounded region of the positive state space. Hence, the solution can be extended for all $t \geq 0$.

3.2 Positivity of Solutions

For the proposed model to have economic significance, its state variables must remain nonnegative throughout the period of analysis. In particular, human capital, physical capital, and technological development cannot assume negative values. We therefore establish that the positive orthant

$$\mathbb{R}^3_+ = \left\{(H, K, T) \in \mathbb{R}^3 : H \geq 0,\ K \geq 0,\ T \geq 0\right\}$$

is positively invariant under system (1).

From the first equation, we have

$$\frac{dH}{dt} + \mu_H H = a_H + \alpha_H T.$$

Using the integrating factor $e^{\mu_H t}$ gives

$$H(t) = H_0 e^{-\mu_H t} + \int_0^t e^{-\mu_H (t-\tau)} \left(a_H + \alpha_H T(\tau)\right) d\tau. \tag{9}$$

Since $H_0 > 0$, $a_H > 0$, and $\alpha_H > 0$, it follows that

$$H(t) > 0, \qquad t \geq 0,$$

provided that $T(t) \geq 0$.

Similarly, the third equation can be expressed as

$$\frac{dT}{dt} + \mu_T T = a_T + \alpha_T H.$$

Multiplying by the integrating factor $e^{\mu_T t}$ and integrating gives

$$T(t) = T_0 e^{-\mu_T t} + \int_0^t e^{-\mu_T (t-\tau)} \left(a_T + \alpha_T H(\tau)\right) d\tau. \tag{10}$$

Since $T_0 > 0$, $a_T > 0$, and $\alpha_T > 0$, we obtain

$$T(t) > 0, \qquad t \geq 0,$$

provided that $H(t) \geq 0$.

For physical capital, the second equation gives

$$\frac{dK}{dt} + \delta K = sAK^{\gamma}H^{\beta}T^{\theta}.$$

Applying the integrating factor $e^{\delta t}$ yields

$$K(t) = K_0 e^{-\delta t} + sA\int_0^t e^{-\delta(t-\tau)}K^{\gamma}(\tau)H^{\beta}(\tau)T^{\theta}(\tau)d\tau. \tag{11}$$

Since all quantities in the integral are nonnegative and $K_0 > 0$, it follows that

$$K(t) > 0, \qquad t \geq 0.$$

Consequently, all three state variables remain strictly positive for all $t \geq 0$. Thus, the positive orthant $\mathbb{R}^3_{++}$ is positively invariant under the proposed model.

3.3 Boundedness of Solutions

We next establish the boundedness of the model solutions. This property ensures that the productive factors do not increase indefinitely and that the model admits economically meaningful long-term behaviour.

Consider the human-capital and technological-development subsystem:

$$\begin{aligned} \frac{dH}{dt} &= a_H + \alpha_H T - \mu_H H, \\ \frac{dT}{dt} &= a_T + \alpha_T H - \mu_T T. \end{aligned} \tag{12}$$

In matrix form, this subsystem can be written as

$$\frac{d}{dt}\begin{pmatrix} H \\ T \end{pmatrix} = \begin{pmatrix} a_H \\ a_T \end{pmatrix} + \begin{pmatrix} -\mu_H & \alpha_H \\ \alpha_T & -\mu_T \end{pmatrix}\begin{pmatrix} H \\ T \end{pmatrix}. \tag{13}$$

The characteristic polynomial of the coefficient matrix is

$$\lambda^2 + (\mu_H + \mu_T)\lambda + \mu_H\mu_T - \alpha_H\alpha_T = 0. \tag{14}$$

By the Routh–Hurwitz criterion, the two eigenvalues of the coefficient matrix have negative real parts if

$$\mu_H\mu_T > \alpha_H\alpha_T. \tag{15}$$

Under condition (15), the human-capital and technological-development subsystem is asymptotically stable. Therefore, $H(t)$ and $T(t)$ remain bounded and converge to their respective equilibrium values. In particular, there exist positive constants M_H and M_T such that

$$0 < H(t) \leq M_H, \qquad 0 < T(t) \leq M_T, \qquad t \geq 0.$$

To establish the boundedness of physical capital, consider

$$\frac{dK}{dt} = sAK^{\gamma}H^{\beta}T^{\theta} - \delta K.$$

Since $H(t)$ and $T(t)$ are bounded, there exists a positive constant

$$M = sAM_H^{\beta}M_T^{\theta}$$

such that

$$\frac{dK}{dt} \leq MK^{\gamma} - \delta K. \tag{16}$$

Because $0 < \gamma < 1$, the term K^{γ} grows more slowly than the linear depreciation term K. Choose

$$K_{\max} = \max\left\{K_0, \left(\frac{2M}{\delta}\right)^{\frac{1}{1-\gamma}}\right\}.$$

Whenever $K \geq K_{\max}$, we have

$$MK^{\gamma} \leq \frac{\delta}{2}K.$$

Consequently,

$$\frac{dK}{dt} \leq -\frac{\delta}{2}K < 0 \qquad \text{whenever } K \geq K_{\max}.$$

Thus, physical capital cannot increase beyond the chosen upper bound. It follows that

$$0 < K(t) \leq K_{\max}, \qquad t \geq 0.$$

Therefore, all solutions of system (12) are bounded whenever

$$\mu_H \mu_T > \alpha_H \alpha_T, \qquad 0 < \gamma < 1.$$

Hence, the proposed model possesses positive and bounded solutions for all $t \geq 0$.

3.4 Equilibrium Analysis

An equilibrium point of system (1) is a point at which the rates of change of all state variables are simultaneously zero. Let the equilibrium point be denoted by

$$E^* = (H^*, K^*, T^*).$$

At equilibrium,

$$\frac{dH}{dt} = \frac{dK}{dt} = \frac{dT}{dt} = 0.$$

Therefore, the equilibrium conditions are

$$\begin{aligned} a_H + \alpha_H T^* - \mu_H H^* &= 0, \\ sA(K^*)^\gamma (H^*)^\beta (T^*)^\theta - \delta K^* &= 0, \\ a_T + \alpha_T H^* - \mu_T T^* &= 0. \end{aligned} \tag{17}$$

Solving the first and third equations simultaneously gives

$$\begin{aligned} \mu_H H^* - \alpha_H T^* &= a_H, \\ -\alpha_T H^* + \mu_T T^* &= a_T. \end{aligned}$$

The determinant of the coefficient matrix is

$$\Delta = \mu_H \mu_T - \alpha_H \alpha_T.$$

Provided that $\Delta > 0$, the equilibrium values of human capital and technological development are

$$H^* = \frac{a_H \mu_T + \alpha_H a_T}{\Delta}, \tag{18}$$

and

$$T^* = \frac{\mu_H a_T + \alpha_T a_H}{\Delta}. \tag{19}$$

Since all parameters in the numerators are positive and $\Delta > 0$, both H^* and T^* are strictly positive. From the second equilibrium equation, we obtain

$$sA(K^*)^\gamma (H^*)^\beta (T^*)^\theta = \delta K^*.$$

Rearranging gives

$$(K^*)^{1-\gamma} = \frac{sA}{\delta}(H^*)^\beta (T^*)^\theta.$$

Since $0 < \gamma < 1$, the equilibrium value of physical capital is

$$K^* = \left[\frac{sA}{\delta}(H^*)^\beta (T^*)^\theta\right]^{\frac{1}{1-\gamma}}. \tag{20}$$

Consequently, the unique positive equilibrium point of the proposed model is

$$E^* = \left(\frac{a_H \mu_T + \alpha_H a_T}{\Delta}, \left[\frac{sA}{\delta}(H^*)^\beta (T^*)^\theta\right]^{\frac{1}{1-\gamma}}, \frac{\mu_H a_T + \alpha_T a_H}{\Delta}\right), \tag{21}$$

where

$$\Delta = \mu_H \mu_T - \alpha_H \alpha_T.$$

The equilibrium exists uniquely in the positive region provided that

$$\mu_H \mu_T > \alpha_H \alpha_T, \qquad 0 < \gamma < 1, \qquad sA > 0.$$

The corresponding equilibrium output is

$$Y^* = A(K^*)^\gamma (H^*)^\beta (T^*)^\theta. \tag{22}$$

3.5 Local Stability Analysis

We now investigate the local asymptotic stability of the positive equilibrium point E^*. The analysis is carried out by evaluating the Jacobian matrix of system (1) at equilibrium.

Let

$$F(H,K,T) = \begin{pmatrix} a_H + \alpha_H T - \mu_H H \\ sAK^\gamma H^\beta T^\theta - \delta K \\ a_T + \alpha_T H - \mu_T T \end{pmatrix}.$$

The Jacobian matrix is given by

$$J(H,K,T) = \begin{pmatrix} -\mu_H & 0 & \alpha_H \\ sA\beta K^\gamma H^{\beta-1} T^\theta & sA\gamma K^{\gamma-1} H^\beta T^\theta - \delta & sA\theta K^\gamma H^\beta T^{\theta-1} \\ \alpha_T & 0 & -\mu_T \end{pmatrix}. \tag{23}$$

Evaluating this matrix at the equilibrium point $E^* = (H^*, K^*, T^*)$ gives

$$J(E^*) = \begin{pmatrix} -\mu_H & 0 & \alpha_H \\ \dfrac{\delta\beta K^*}{H^*} & -\delta(1-\gamma) & \dfrac{\delta\theta K^*}{T^*} \\ \alpha_T & 0 & -\mu_T \end{pmatrix}. \tag{24}$$

In obtaining the second row, we have used the equilibrium identity

$$sA(K^*)^\gamma (H^*)^\beta (T^*)^\theta = \delta K^*.$$

The characteristic equation is obtained from

$$\det(\lambda I - J(E^*)) = 0.$$

Expanding the determinant gives

$$[\lambda + \delta(1-\gamma)]\left[\lambda^2 + (\mu_H + \mu_T)\lambda + \mu_H\mu_T - \alpha_H\alpha_T\right] = 0. \tag{25}$$

Therefore, the eigenvalues are

$$\lambda_1 = -\delta(1-\gamma), \tag{26}$$

and

$$\lambda_{2,3} = -\frac{\mu_H + \mu_T}{2} \pm \frac{1}{2}\sqrt{(\mu_H - \mu_T)^2 + 4\alpha_H\alpha_T}. \tag{27}$$

Since $\delta > 0$ and $0 < \gamma < 1$, it follows that

$$\lambda_1 < 0.$$

Furthermore, under the condition

$$\mu_H\mu_T > \alpha_H\alpha_T,$$

we have

$$(\mu_H + \mu_T)^2 > (\mu_H - \mu_T)^2 + 4\alpha_H\alpha_T.$$

Consequently,

$$\lambda_2 < 0, \qquad \lambda_3 < 0.$$

Thus, all eigenvalues of the Jacobian matrix have negative real parts. By the standard linearisation theorem for ordinary differential equations, the positive equilibrium E^* is locally asymptotically stable.

We therefore establish the following result.

Theorem. Suppose that all model parameters are positive and

$$0 < \gamma < 1, \qquad \beta, \theta > 0, \qquad \gamma + \beta + \theta = 1.$$

If

$$\mu_H\mu_T > \alpha_H\alpha_T,$$

then system (1) possesses a unique positive equilibrium point E^*, which is locally asymptotically stable.

Proof. The existence and uniqueness of the positive equilibrium follow from the equilibrium expressions (18)–(20). The characteristic equation of the Jacobian matrix at equilibrium is given by (25). Under the stated conditions, all three eigenvalues have negative real parts. Therefore, the equilibrium is locally asymptotically stable. This completes the proof. □

3.6 Economic Interpretation of the Stability Result

This stability condition is economically significant for the long-term development of Imo State. The result $\mu_H\mu_T > \alpha_H\alpha_T$, implies that the combined obsolescence and depreciation rates are dominant over the reinforcing interaction between human capital and technological development.

The parameters α_H and α_T capture the reinforcing contributions of technology to human capital development and human capital to technological development, respectively. Although these feedbacks can drive economic development, if the interaction is too strong compared to the depreciation effects, the human-capital and technology subsystem may never approach a stable equilibrium.

The condition $0 < \gamma < 1$ assures that the investment generated by output increase due to a high capital stock is less than the linear depreciation term when capital stock becomes large. Thus, the physical-capital stock is bounded, and has a finite positive equilibrium.

The local asymptotic stability of E^* means that economic paths starting close to the equilibrium will converge to it as time increases. In other words, small fluctuations in human capital, physical capital, or technological development will eventually die out, and the system will approach the long-run equilibrium state.

Thus, the result implies that sustained economic growth in the proposed model depends on a balance between productive-factor accumulation, human-capital, and technological development, on the one hand, and obsolescence and depreciation, on the other.

4 Discussion

This research proposes a deterministic mathematical model to analyze the dynamical relationship between human capital, physical capital, technological development, and economic growth in Imo State, Nigeria. The model is formulated as a system of ordinary differential equations. Human capital, physical capital, and technological development are modeled as three interacting state variables that change with time, while total economic output is modeled as an algebraic output variable, using a Cobb-Douglas production function. This model presents a mathematical formulation of how the accumulation of productive assets, as well as their interaction with each other, can affect the long-term economic growth path of the state.

A major component of the model formulation is the interaction between human capital and technological development. Human capital stimulates technological development through the term $\alpha_T H$, while technological development encourages human capital formation through the term $\alpha_H T$. This is in line with the observed relationship between education, skills acquisition, technological adoption, and innovation as drivers of economic development. In the context of Imo State, an improvement in education and training will lead to a more productive labor force. At the same time, a more knowledgeable labor force will be able to effectively utilize technological advances. The proposed model therefore incorporates the potential for a bidirectional relationship between human-capital development and technological advancement.

The physical-capital equation adds another dimension to the model. Physical capital is accumulated through investment, represented by a proportion s of total economic output, while it is depleted at a rate δ. There is a feedback relationship between economic output and capital investment. An increase in economic output leads to higher investment and therefore a greater physical-capital stock, while the physical capital stock leads to higher economic output through the Cobb-Douglas production function. This emphasizes the importance of investment in machines, factories, roads, ports, airports, and other physical infrastructure that can enhance the productivity of Imo State. However, the physical capital accumulation process will be effective only if investment is higher than depreciation.

The qualitative analysis of the model proved the positivity of solutions of the system of equations under positive initial conditions and positive model parameter values. This is an economically significant property since human capital, physical capital, and technological development are productive assets that can be modeled as positive state variables within the proposed formulation. Furthermore, boundedness was proved for solutions of the model, as long as $\mu_H\mu_T > \alpha_H\alpha_T$ and $0 < \gamma < 1$. The first of these two conditions prevents the bidirectional encouragement of human capital and technological development from resulting in an unbounded solution, while the second prevents the physical-capital accumulation equation from producing solutions that increase without bound when the physical capital stock is large in comparison to its linear depreciation term.

The analysis of equilibrium yields a unique positive equilibrium if and only if $\mu_H\mu_T > \alpha_H\alpha_T$, and in addition, the parameters corresponding to investment, productivity and output elasticity are positive and satisfy $0 < \gamma < 1$. The equilibria are found by setting the time-derivatives of human capital, physical capital and technological development to zero. Then the equilibria corresponding to the human capital and technological development variables are proportional to the corresponding base production levels, and are determined by their own depreciation rates and the magnitude of the bidirectional encouragement.

Most importantly, the denominator $\Delta = \mu_H\mu_T - \alpha_H\alpha_T$ has to be positive, which is equivalent to the determinant condition of the equilibrium physical capital stock that is given explicitly by the investment rate, total factor productivity, physical-capital depreciation, and the equilibrium levels of human capital and technology.

Furthermore, the local stability analysis showed that the positive equilibrium is locally asymptotically stable when $\mu_H\mu_T > \alpha_H\alpha_T$ and $0 < \gamma < 1$. The characteristic equation of the linearized system can be factored as a term describing the physical capital dynamics, and a quadratic which describes the relationship between human capital and technological development. The physical-capital eigenvalue is negative, since the depreciation rate is positive and the physical capital output elasticity is less than one. The real part of the other two eigenvalues is negative since the determinant $\mu_H\mu_T - \alpha_H\alpha_T$ is positive by assumption in the theorem, so the solutions of the model system approach the positive equilibrium point for positive initial populations. Economically, this result means that the proposed formulation is robust enough to return to a long-term equilibrium after a small perturbation of one of the state variables.

The economic interpretation of the constant-returns-to-scale assumption that was made in the production function is also worth mentioning; the equality $\gamma + \beta + \theta = 1$ implies that a proportional increase in physical capital, human capital and technological development will cause a proportional increase in economic output. This is a very convenient assumption to work with when analyzing the effects of the three productive assets on economic growth, but the actual values of the three output elasticities must be obtained from the parameter estimation process.

In the context of economic development in Imo State, the proposed model formulation emphasizes the role of bidirectional relationships between the productive assets, rather than the traditional approach of examining the effect of capital accumulation on output. Economic growth cannot be evaluated purely from the point of view of capital and output; instead, the interactions between human capital and technological development must be considered. Investment in education, technical and vocational training, technology adoption, and physical infrastructure may all lead to a sustainable economic growth process, as is implied by the model formulation.

Of course, there are limitations in this formulation. The model parameters are assumed to be constant with respect to time, while realistically, investment, depreciation, and technological development are subject to constant change. Furthermore, government fiscal policy, inflation, unemployment, population growth, political instability, and other factors influencing economic development are not represented explicitly in this formulation. Also, this is a deterministic model and does not incorporate uncertainty. Human capital and technological development are modeled as aggregate variables, neglecting the differences in educational status and technological advancement that may exist between economic activity sectors and industries, as well as within the same group of people.

Overall, it was established that the model has a positive and bounded solution trajectory under the conditions $\mu_H\mu_T > \alpha_H\alpha_T$ and $0 < \gamma < 1$. A unique positive equilibrium point was found under the same conditions, and solutions to the system were shown to approach the equilibrium point, implying that the system is stable under these assumptions. This analysis provides a good foundation for the continued mathematical investigation of economic growth in Imo State, Nigeria.

5 Conclusion

This research proposes a deterministic mathematical model to study the interaction among human capital, physical capital, technological development and economic growth in Imo State, Nigeria. The model is formulated as a system of three ordinary differential equations that represents the human capital, physical capital and technological development as three dependent variables, together with a Cobb-Douglas production function as an independent variable for economic growth. The formulation takes into account the effect of technological development on human capital, that of human capital on technological development and the investment from total economic output in physical capital.

The qualitative properties of the proposed model are discussed in terms of existence and uniqueness of solutions, positivity, boundedness, equilibrium and local asymptotic stability. Existence and uniqueness of solutions is established for positive solutions with positive initial conditions. Boundedness is further established for this class of solutions assuming $\mu_H\mu_T > \alpha_H\alpha_T$ and $0 < \gamma < 1$, meaning that the joint effect of human capital and technological development on each other is not more pronounced than the effect of depreciation on the independent growth of these state variables and the rate at which physical capital is consumed over time does not exceed that at which it is replenished through investment.

The equilibrium analysis gives one positive equilibrium point whose elements are explicitly defined in terms of the parameters of the model, for instance the equilibrium value of human capital depends on its own baseline rate, the baseline rate of technological development, the depreciation rate and their joint effect, the equilibrium value of technological development depends on its own baseline rate, the

baseline rate of human capital, the depreciation rate and their joint effect, while the equilibrium value of physical capital depends on the investment rate, the total factor productivity, the depreciation rate and the equilibrium values of human capital and technological development. The local asymptotic stability of the equilibrium point is established to hold when $\mu_H\mu_T > \alpha_H\alpha_T$ and $0 < \gamma < 1$, in which case, any small perturbation around the equilibrium is damped out with time.

The results show that economic growth is significantly influenced by the productive capacity represented by human capital and technological development, both of which are supported by and in turn support each other, while the physical capital available in the system is governed by the investment rate and the overall level of economic growth. The model therefore, describes a closed system in which human capital and technological development represent the major growth factors that generate the output and sustain a stable steady state level of all economic indicators, if certain assumptions about the relationship between the parameters hold.

The model studied in this work can be extended in future by including other major economic indicators, such as government expenditure, population growth, unemployment, infrastructural development and foreign investment, or can be made more realistic by estimating the parameters from an economic database on the state of Imo.

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