

An application of absolute matrix summability to infinite series

ABSTRACT. In this paper, a known result concerning the absolute Riesz summability factors of an infinite series is extended to the more general framework of absolute matrix summability and sufficient conditions for the $\varphi\text{-}|B; \delta|_k$ summability of an infinite series are established.

Keywords: summability factors, infinite series, absolute matrix summability, Minkowski's inequality, Riesz mean, Hölder's inequality.

Mathematics Subject Classification (2020): 26D15, 40D15, 40F05, 40G99.

1 Introduction

A positive sequence (h_η) is called *almost increasing* if there exist a positive increasing sequence (a_η) together with positive constants K and L satisfying, (see [1])

$$Ka_\eta \leq h_\eta \leq La_\eta$$

It is clear that every increasing sequence is also almost increasing, whereas the converse does not necessarily hold. For example, the sequence $h_\eta = \eta e^{(-1)^\eta}$ is almost increasing but is not increasing.

For the infinite series $\sum d_\eta$, let (s_η) represent the corresponding sequence of partial sums. Suppose that $B = (b_{\eta\omega})$ is a normal matrix, that is, a lower triangular matrix whose diagonal entries are all nonzero. The matrix B induces the sequence transformation $s = (s_\eta) \mapsto Bs = (B_\eta(s))$, where

$$B_\eta(s) = \sum_{\omega=0}^{\eta} b_{\eta\omega} s_\omega, \quad \eta = 0, 1, \dots$$

Consider the positive sequence (φ_η) . The infinite series $\sum d_\eta$ is $\varphi\text{-}|B;\delta|_k$ summable, provided that [8]

$$\sum_{\eta=1}^{\infty} \varphi_\eta^{\delta k + k - 1} |B_\eta(s) - B_{\eta-1}(s)|^k < \infty, \quad k \geq 1 \quad \text{and} \quad \delta \geq 0.$$

The infinite series $\sum d_\eta$ is called $|\bar{N}, q_\eta|_k$ summable, if

$$\sum_{\eta=1}^{\infty} \left(\frac{Q_\eta}{q_\eta} \right)^{k-1} |\alpha_\eta - \alpha_{\eta-1}|^k < \infty, \quad k \geq 1 \quad (\text{see [2]})$$

where

$$\alpha_\eta = \frac{1}{Q_\eta} \sum_{\omega=0}^{\eta} q_\omega s_\omega,$$

and (q_η) is a positive sequence satisfying

$$Q_\eta = \sum_{\omega=0}^{\eta} q_\omega \longrightarrow \infty, \quad \eta \rightarrow \infty \quad (Q_{-\omega} = q_{-\omega} = 0, \omega \geq 1).$$

If $\varphi_\eta = \frac{Q_\eta}{q_\eta}$ and $\delta = 0$, then the $\varphi\text{-}|B;\delta|_k$ summability method coincides with the $|B, q_\eta|_k$ summability method (see [15]). Furthermore, by taking $\varphi_\eta = \frac{Q_\eta}{q_\eta}$, $\delta = 0$ and $b_{\eta\omega} = \frac{q_\omega}{Q_\eta}$, one obtains the $|\bar{N}, q_\eta|_k$ summability method.

2 Known Result

The result below, established in [5], characterizes the $|\bar{N}, q_\eta|_k$ summability factors of an infinite series.

Theorem 1 *Suppose that (X_η) is an almost increasing sequence and that the following conditions are satisfied:*

$$|\xi_\mu| X_\mu = O(1) \quad \text{as} \quad \mu \rightarrow \infty, \tag{1}$$

$$\sum_{\eta=1}^{\mu} \eta X_\eta |\Delta^2 \xi_\eta| = O(1) \quad \text{as} \quad \mu \rightarrow \infty, \tag{2}$$

$$\sum_{\eta=1}^{\mu} \frac{Q_\eta}{\eta} = O(P_\mu) \quad \text{as} \quad \mu \rightarrow \infty, \tag{3}$$

$$\sum_{\eta=1}^{\mu} \frac{|z_{\eta}|^k}{\eta} = O(X_{\mu}) \quad \text{as } \mu \rightarrow \infty, \quad (4)$$

and

$$\sum_{\eta=1}^{\mu} \frac{q_{\eta}}{Q_{\eta}} |z_{\eta}|^k = O(X_{\mu}) \quad \text{as } \mu \rightarrow \infty \quad (5)$$

where $\Delta^2 \xi_{\eta} = \Delta(\Delta \xi_{\eta})$, $\Delta \xi_{\eta} = \xi_{\eta} - \xi_{\eta+1}$, and (z_{η}) is the sequence of $(C, 1)$ means associated with (ηd_{η}) . Under these assumptions, the series $\sum d_{\eta} \xi_{\eta}$ is $|\bar{N}, q_{\eta}|_k$ summable for $k \geq 1$.

3 Main Result

For related results on the absolute matrix summability factors of an infinite series, the reader is referred to [3, 4, 7, 9–14]. The main objective of the present paper is to extend Theorem 1 to the more general setting of φ - $|B; \delta|_k$ summability.

To formulate our main theorem, we first introduce some additional notation. For a given normal matrix $B = (b_{n\omega})$, define the two lower semimatrices $\bar{B} = (\bar{b}_{\eta\omega})$ and $\hat{B} = (\hat{b}_{\eta\omega})$ by

$$\bar{b}_{\eta\omega} = \sum_{v=\omega}^{\eta} b_{\eta v}, \quad \eta, \omega = 0, 1, \dots \quad (6)$$

and

$$\hat{b}_{00} = \bar{b}_{00} = b_{00}, \quad \hat{b}_{\eta\omega} = \bar{b}_{\eta\omega} - \bar{b}_{\eta-1, \omega}, \quad \eta = 1, 2, \dots \quad (7)$$

The transformations from series to sequences and from series to series are given by the matrices $\bar{B}$ and $\hat{B}$, respectively. Hence,

$$B_{\eta}(s) = \sum_{\omega=0}^{\eta} b_{\eta\omega} s_{\omega} = \sum_{\omega=0}^{\eta} \bar{b}_{\eta\omega} a_{\omega} \quad (8)$$

and

$$\bar{\Delta} B_{\eta}(s) = \sum_{\omega=0}^{\eta} \hat{b}_{\eta\omega} a_{\omega}. \quad (9)$$

Theorem 2 Consider a positive normal matrix $B = (b_{\eta\omega})$ satisfying

$$\bar{b}_{\eta 0} = 1, \quad \eta = 0, 1, \dots, \quad (10)$$

$$b_{\eta-1, \omega} \geq b_{\eta\omega}, \quad \text{for } \eta \geq \omega + 1, \quad (11)$$

$$b_{\eta\eta} = O\left(\frac{q_\eta}{Q_\eta}\right), \quad (12)$$

$$\sum_{\omega=1}^{\eta-1} \frac{|\hat{b}_{\eta,\omega+1}|}{\omega} = O(b_{\eta\eta}), \quad (13)$$

$$\sum_{\eta=\omega+1}^{\mu+1} \varphi_\eta^{\delta k} |\hat{b}_{\eta,\omega+1}| = O(\varphi_\omega^{\delta k}) \quad \text{as } \mu \rightarrow \infty, \quad (14)$$

$$\sum_{\eta=\omega+1}^{\mu+1} \varphi_\eta^{\delta k} |\Delta_\omega \hat{b}_{\eta\omega}| = O(\varphi_\omega^{\delta k-1}) \quad \text{as } \mu \rightarrow \infty, \quad (15)$$

where $\Delta_\omega(\hat{b}_{\eta\omega}) = \hat{b}_{\eta\omega} - \hat{b}_{\eta,\omega+1}$. Assume that (X_η) is an almost increasing sequence satisfying $\varphi_\eta q_\eta = O(Q_\eta)$. Furthermore, suppose that conditions (1) and (2) of Theorem 1, together with

$$\sum_{\eta=1}^{\mu} \varphi_\eta^{\delta k} \frac{1}{\eta} |z_\eta|^k = O(X_\mu) \quad \text{as } \mu \rightarrow \infty, \quad (16)$$

$$\sum_{\eta=1}^{\mu} \varphi_\eta^{\delta k-1} |z_\eta|^k = O(X_\mu) \quad \text{as } \mu \rightarrow \infty \quad (17)$$

hold. Under these assumptions, the series $\sum d_\eta \xi_\eta$ is φ - $|B; \delta|_k$ summable for $0 \leq \delta < 1/k$ and $k \geq 1$.

Before proving Theorem 2, we state the following lemma.

Lemma 1 ([5]) *Whenever the conditions of Theorem 2 hold, it follows that*

$$\eta X_\eta |\Delta \xi_\eta| = O(1) \quad \text{as } \eta \rightarrow \infty, \quad (18)$$

$$\sum_{\eta=1}^{\infty} X_\eta |\Delta \xi_\eta| < \infty. \quad (19)$$

4 Proof of Theorem 2

Let the B -transform of the series $\sum d_\eta \xi_\eta$ be (θ_η) . Using (8) and (9), we obtain

$$\bar{\Delta} \theta_\eta = \sum_{\omega=1}^{\eta} \frac{\hat{b}_{\eta\omega} \xi_\omega}{\omega} \omega d_\omega.$$

Using Abel's formula, we get

$$\begin{aligned}
 \bar{\Delta}\theta_\eta &= \sum_{\omega=1}^{\eta-1} \Delta_\omega \left(\frac{\hat{b}_{\eta\omega}\xi_\omega}{\omega} \right) \sum_{r=1}^{\omega} r d_r + \frac{\hat{b}_{\eta\eta}\xi_\eta}{\eta} \sum_{r=1}^{\eta} r d_r \\
 &= \frac{\eta+1}{\eta} b_{\eta\eta}\xi_\eta z_\eta + \sum_{\omega=1}^{\eta-1} \frac{\omega+1}{\omega} \Delta_\omega \left(\hat{b}_{\eta\omega} \right) \xi_\omega z_\omega \\
 &\quad + \sum_{\omega=1}^{\eta-1} \frac{\omega+1}{\omega} \hat{b}_{\eta,\omega+1} \Delta \xi_\omega z_\omega + \sum_{\omega=1}^{\eta-1} \frac{1}{\omega} \hat{b}_{\eta,\omega+1} \xi_{\omega+1} z_\omega \\
 &= \theta_{\eta,1} + \theta_{\eta,2} + \theta_{\eta,3} + \theta_{\eta,4}.
 \end{aligned}$$

To prove Theorem 2, we will show $\sum_{\eta=1}^{\infty} \varphi_\eta^{\delta k+k-1} |\theta_{\eta,r}|^k < \infty$ for $r = 1, 2, 3, 4$. First, by using (12), we obtain

$$\begin{aligned}
 \sum_{\eta=1}^{\mu} \varphi_\eta^{\delta k+k-1} |\theta_{\eta,1}|^k &= \sum_{\eta=1}^{\mu} \varphi_\eta^{\delta k+k-1} \left| \frac{\eta+1}{\eta} b_{\eta\eta}\xi_\eta z_\eta \right|^k \\
 &= O(1) \sum_{\eta=1}^{\mu} \varphi_\eta^{\delta k+k-1} b_{\eta\eta}^k |\xi_\eta|^k |z_\eta|^k \\
 &= O(1) \sum_{\eta=1}^{\mu} \varphi_\eta^{\delta k-1} \left(\frac{\varphi_\eta q_\eta}{Q_\eta} \right)^k |\xi_\eta|^{k-1} |\xi_\eta| |z_\eta|^k.
 \end{aligned}$$

Since (X_η) is an almost increasing sequence, condition (1) implies that $|\xi_\eta|^{k-1} = O(1)$. Hence,

$$\begin{aligned}
 \sum_{\eta=1}^{\mu} \varphi_\eta^{\delta k+k-1} |\theta_{\eta,1}|^k &= O(1) \sum_{\eta=1}^{\mu} \varphi_\eta^{\delta k-1} |\xi_\eta| |z_\eta|^k \\
 &= O(1) \sum_{\eta=1}^{\mu-1} \Delta |\xi_\eta| \sum_{\omega=1}^{\eta} \varphi_\omega^{\delta k-1} |z_\omega|^k + O(1) |\xi_\mu| \sum_{\eta=1}^{\mu} \varphi_\eta^{\delta k-1} |z_\eta|^k \\
 &= O(1) \sum_{\eta=1}^{\mu-1} |\Delta \xi_\eta| X_\eta + O(1) |\xi_\mu| X_\mu = O(1) \quad \text{as } \mu \rightarrow \infty,
 \end{aligned}$$

by means of Abel's transformation together with conditions (17), (19), and (1).

Next, by an application of Hölder's inequality

$$\begin{aligned}
 \sum_{\eta=2}^{\mu+1} \varphi_\eta^{\delta k+k-1} |\theta_{\eta,2}|^k &= \sum_{\eta=2}^{\mu+1} \varphi_\eta^{\delta k+k-1} \left| \sum_{\omega=1}^{\eta-1} \frac{\omega+1}{\omega} \Delta_\omega \left(\hat{b}_{\eta\omega} \right) \xi_\omega z_\omega \right|^k \\
 &= O(1) \sum_{\eta=2}^{\mu+1} \varphi_\eta^{\delta k+k-1} \left(\sum_{\omega=1}^{\eta-1} \left| \Delta_\omega \left(\hat{b}_{\eta\omega} \right) \right| |\xi_\omega| |z_\omega| \right)^k \\
 &= O(1) \sum_{\eta=2}^{\mu+1} \varphi_\eta^{\delta k+k-1} \sum_{\omega=1}^{\eta-1} \left| \Delta_\omega \left(\hat{b}_{\eta\omega} \right) \right| |\xi_\omega|^k |z_\omega|^k \left(\sum_{\omega=1}^{\eta-1} \left| \Delta_\omega \left(\hat{b}_{\eta\omega} \right) \right| \right)^{k-1}.
 \end{aligned}$$

Here using (6), (7), (10) and (11), it is clear that

$$\sum_{\omega=1}^{\eta-1} |\Delta_{\omega}(\hat{b}_{\eta\omega})| = \sum_{\omega=1}^{\eta-1} (b_{\eta-1,\omega} - b_{\eta\omega}) \leq b_{\eta\eta}.$$

Now, using (12), (15) and (1),

$$\begin{aligned} \sum_{\eta=2}^{\mu+1} \varphi_{\eta}^{\delta k+k-1} |\theta_{\eta,2}|^k &= O(1) \sum_{\eta=2}^{\mu+1} \varphi_{\eta}^{\delta k+k-1} b_{\eta\eta}^{k-1} \left(\sum_{\omega=1}^{\eta-1} |\Delta_{\omega}(\hat{b}_{\eta\omega})| |\xi_{\omega}|^k |z_{\omega}|^k \right) \\ &= O(1) \sum_{\eta=2}^{\mu+1} \varphi_{\eta}^{\delta k} \left(\frac{\varphi_{\eta} q_{\eta}}{Q_{\eta}} \right)^{k-1} \left(\sum_{\omega=1}^{\eta-1} |\Delta_{\omega}(\hat{b}_{\eta\omega})| |\xi_{\omega}|^k |z_{\omega}|^k \right) \\ &= O(1) \sum_{\omega=1}^{\mu} |\xi_{\omega}|^k |z_{\omega}|^k \sum_{\eta=\omega+1}^{\mu+1} \varphi_{\eta}^{\delta k} |\Delta_{\omega}(\hat{b}_{\eta\omega})| \\ &= O(1) \sum_{\omega=1}^{\mu} \varphi_{\omega}^{\delta k-1} |\xi_{\omega}| |z_{\omega}|^k. \end{aligned}$$

Proceeding as in the estimate for $\theta_{\eta,1}$,

$$\sum_{\eta=2}^{\mu+1} \varphi_{\eta}^{\delta k+k-1} |\theta_{\eta,2}|^k = O(1) \text{ as } \mu \rightarrow \infty.$$

A further application of Hölder's inequality gives

$$\begin{aligned} \sum_{\eta=2}^{\mu+1} \varphi_{\eta}^{\delta k+k-1} |\theta_{\eta,3}|^k &= \sum_{\eta=2}^{\mu+1} \varphi_{\eta}^{\delta k+k-1} \left| \sum_{\omega=1}^{\eta-1} \frac{\omega+1}{\omega} \hat{b}_{\eta,\omega+1} \Delta \xi_{\omega} z_{\omega} \right|^k \\ &= O(1) \sum_{\eta=2}^{\mu+1} \varphi_{\eta}^{\delta k+k-1} \left(\sum_{\omega=1}^{\eta-1} |\hat{b}_{\eta,\omega+1}| |\Delta \xi_{\omega}| |z_{\omega}| \right)^k \\ &= O(1) \sum_{\eta=2}^{\mu+1} \varphi_{\eta}^{\delta k+k-1} \sum_{\omega=1}^{\eta-1} |\hat{b}_{\eta,\omega+1}| |\Delta \xi_{\omega}| |z_{\omega}|^k \left(\sum_{\omega=1}^{\eta-1} |\hat{b}_{\eta,\omega+1}| |\Delta \xi_{\omega}| \right)^{k-1}. \end{aligned}$$

Here

$$\begin{aligned}\hat{b}_{\eta,\omega+1} &= \bar{b}_{\eta,\omega+1} - \bar{b}_{\eta-1,\omega+1} \\ &= b_{\eta\eta} + \sum_{v=\omega+1}^{\eta-1} (b_{\eta v} - b_{\eta-1,v}) \leq b_{\eta\eta}\end{aligned}$$

by (7), (6) and (11).

And also using the condition (19), we get

$$\sum_{\eta=2}^{\mu+1} \varphi_{\eta}^{\delta k+k-1} |\theta_{\eta,3}|^k = O(1) \sum_{\eta=2}^{\mu+1} \varphi_{\eta}^{\delta k+k-1} b_{\eta\eta}^{k-1} \left(\sum_{\omega=1}^{\eta-1} |\hat{b}_{\eta,\omega+1}| |\Delta \xi_{\omega}| |z_{\omega}|^k \right).$$

Then using (12), (14), (16), (2), (19) and (18), we obtain

$$\begin{aligned}\sum_{\eta=2}^{\mu+1} \varphi_{\eta}^{\delta k+k-1} |\theta_{\eta,3}|^k &= O(1) \sum_{\eta=2}^{\mu+1} \varphi_{\eta}^{\delta k} \left(\frac{\varphi_{\eta} q_{\eta}}{Q_{\eta}} \right)^{k-1} \left(\sum_{\omega=1}^{\eta-1} |\hat{b}_{\eta,\omega+1}| |\Delta \xi_{\omega}| |z_{\omega}|^k \right) \\ &= O(1) \sum_{\omega=1}^{\mu} |\Delta \xi_{\omega}| |z_{\omega}|^k \sum_{\eta=\omega+1}^{\mu+1} \varphi_{\eta}^{\delta k} |\hat{b}_{\eta,\omega+1}| \\ &= O(1) \sum_{\omega=1}^{\mu} \varphi_{\omega}^{\delta k} \omega |\Delta \xi_{\omega}| \frac{|z_{\omega}|^k}{\omega} \\ &= O(1) \sum_{\omega=1}^{\mu-1} \Delta(\omega |\Delta \xi_{\omega}|) \sum_{r=1}^{\omega} \varphi_r^{\delta k} \frac{1}{r} |z_r|^k + O(1) \mu |\Delta \xi_{\mu}| \sum_{\omega=1}^{\mu} \varphi_{\omega}^{\delta k} \frac{1}{\omega} |z_{\omega}|^k \\ &= O(1) \sum_{\omega=1}^{\mu-1} \omega X_{\omega} |\Delta^2 \xi_{\omega}| + O(1) \sum_{\omega=1}^{\mu-1} |\Delta \xi_{\omega}| X_{\omega} + O(1) \mu |\Delta \xi_{\mu}| X_{\mu} \\ &= O(1) \quad as \quad \mu \rightarrow \infty.\end{aligned}$$

As a final step, we derive

$$\begin{aligned}
\sum_{\eta=2}^{\mu+1} \varphi_{\eta}^{\delta k+k-1} |\theta_{\eta,4}|^k &= \sum_{\eta=2}^{\mu+1} \varphi_{\eta}^{\delta k+k-1} \left| \sum_{\omega=1}^{\eta-1} \frac{1}{\omega} \hat{b}_{\eta,\omega+1} \xi_{\omega+1} z_{\omega} \right|^k \\
&\leq \sum_{\eta=2}^{\mu+1} \varphi_{\eta}^{\delta k+k-1} \left(\sum_{\omega=1}^{\eta-1} |\hat{b}_{\eta,\omega+1}| |\xi_{\omega+1}| \frac{|z_{\omega}|}{\omega} \right)^k \\
&\leq \sum_{\eta=2}^{\mu+1} \varphi_{\eta}^{\delta k+k-1} \left(\sum_{\omega=1}^{\eta-1} |\hat{b}_{\eta,\omega+1}| |\xi_{\omega+1}|^k \frac{|z_{\omega}|^k}{\omega} \right) \left(\sum_{\omega=1}^{\eta-1} \frac{|\hat{b}_{\eta,\omega+1}|}{\omega} \right)^{k-1} \\
&= O(1) \sum_{\eta=2}^{\mu+1} \varphi_{\eta}^{\delta k+k-1} b_{\eta\eta}^{k-1} \left(\sum_{\omega=1}^{\eta-1} |\hat{b}_{\eta,\omega+1}| |\xi_{\omega+1}| \frac{|z_{\omega}|^k}{\omega} \right) \\
&= O(1) \sum_{\eta=2}^{\mu+1} \varphi_{\eta}^{\delta k} \left(\frac{\varphi_{\eta} q_{\eta}}{Q_{\eta}} \right)^{k-1} \left(\sum_{\omega=1}^{\eta-1} |\hat{b}_{\eta,\omega+1}| |\xi_{\omega+1}| \frac{|z_{\omega}|^k}{\omega} \right) \\
&= O(1) \sum_{\omega=1}^{\mu} |\xi_{\omega+1}| \frac{|z_{\omega}|^k}{\omega} \sum_{\eta=\omega+1}^{\mu+1} \varphi_{\eta}^{\delta k} |\hat{b}_{\eta,\omega+1}| \\
&= O(1) \sum_{\omega=1}^{\mu} \varphi_{\omega}^{\delta k} \frac{1}{\omega} |\xi_{\omega+1}| |z_{\omega}|^k \\
&= O(1) \sum_{\omega=1}^{\mu-1} |\Delta \xi_{\omega+1}| \sum_{r=1}^{\omega} \varphi_r^{\delta k} \frac{1}{r} |z_r|^k + O(1) |\xi_{\mu+1}| \sum_{\omega=1}^{\mu} \varphi_{\omega}^{\delta k} \frac{1}{\omega} |z_{\omega}|^k \\
&= O(1) \sum_{\omega=1}^{\mu-1} |\Delta \xi_{\omega+1}| X_{\omega+1} + O(1) |\xi_{\mu+1}| X_{\mu+1} \\
&= O(1) \quad \text{as } \mu \rightarrow \infty,
\end{aligned}$$

by using (13), (12), (14), (16), (19) and (1). Accordingly, Theorem 2 is proved.

5 Conclusions

A direct consequence of Theorem 2 is obtained by choosing $\delta = 0$ and $\varphi_{\eta} = \frac{Q_{\eta}}{q_{\eta}}$, yielding the result on $|B, q_{\eta}|_k$ summability established in (see [6]). Furthermore, if $\delta = 0$, $\varphi_{\eta} = \frac{Q_{\eta}}{q_{\eta}}$ and $b_{\eta\omega} = \frac{q_{\omega}}{Q_{\eta}}$ Theorem 2 reduces to Theorem 1.

This manuscript was prepared through the intellectual contributions of the author(s) to the study design, data analysis, interpretation of results, and scholarly content. Grammarly and ChatGPT were used solely to assist with grammatical refinement and reference formatting during manuscript revision. The author(s) accept full responsibility for the accuracy, integrity, and final content of the manuscript.

References

- [1] Bari, N. K., Stechkin, S. B. Best approximations and differential properties of two conjugate functions. *Trudy Moskov. Mat. Obšč.* 5 (1956): 483-522 (in Russian).
- [2] Bor, H. On two summability methods. *Math. Proc. Cambridge Philos. Soc.* 97(1) (1985): 147-149.
- [3] Karakaş, A. On absolute matrix summability factors of infinite series. *J. Class. Anal.* 13(2) (2018): 133-139.
- [4] Kartal, B. On an extension of absolute summability. *Konuralp J. Math.* 7(2) (2019): 433-437.
- [5] Mazhar, S. M. Absolute summability factors of infinite series. *Kyungpook Math. J.* 39(1) (1999): 67-73.
- [6] Ögdük, N. A summability factor theorem by using an almost increasing sequence. *J. Comp. Anal. Appl.* 11(1) (2009): 45-53.
- [7] Özarslan, H. S., Şakar, M. Ö. A new application of absolute matrix summability. *Math. Sci. Appl. E-Notes* 3(1) (2015): 36-43.
- [8] Özarslan, H. S. A new study on generalised absolute matrix summability methods. *Maejo Int. J. Sci. Technol.* 12(3) (2018): 199-205.
- [9] Özarslan, H. S. A new factor theorem for absolute matrix summability. *Quaest. Math.* 42(6) (2019): 803-809.
- [10] Özarslan, H. S. “An application of absolute matrix summability using almost increasing and δ -quasi-monotone sequences.” *Kyungpook Math. J.* 59(2) (2019): 233-240.
- [11] Özarslan, H. S., Kartal, B. Absolute matrix summability via almost increasing sequence. *Quaest. Math.* 43(10) (2020): 1477-1485.
- [12] Özarslan, H. S. Generalized almost increasing sequences. *Lobachevskii J. Math.* 42(1) (2021): 167-172.
- [13] Özarslan, H. S. A new theorem on generalized absolute matrix summability. *Azerb. J. Math.* 13(1) (2023): 3–13.

- [14] Özarslan, H. S., Kartal Erdoğan, B. Generalized absolute matrix summability factors. *Publ. Inst. Math. (Beograd) (N.S.)* 117(131) (2025): 77–82.
- [15] Sulaiman, W. T. Inclusion theorems for absolute matrix summability methods of an infinite series IV. *Indian J. Pure Appl. Math.* 34(11) (2003): 1547–1557.