Some results on Generalized Fractional Calculus Operators Associated with the Product of the general class of Polynomials and extended Mittag-Leffler type Function

Abstract

This study evaluates four image formulae involving generalized fractional calculus operators, comprising left-sided and right-sided fractional integration and fractional differentiation. The operators are applied to the product of a general class of polynomials, also known as Srivastava polynomials, and an extended Mittag-Leffler type function. The derivations use the defining series representations of the functions together with established fractional-calculus formulae for power functions. The resulting expressions are represented in terms of the generalized Wright hypergeometric function. For the generalized fractional integration operators, two principal image formulae are obtained for the left-sided and right-sided cases, and corresponding results are developed for generalized fractional differentiation. A sequence of corollaries follows by suitable specialisation of the parameters and the coefficient sequence, including reductions associated with extended and Prabhakar-type Mittag-Leffler functions, Saigo fractional calculus operators, and Riemann-Liouville fractional calculus operators. Because the Srivastava polynomial family contains several familiar polynomial systems as special cases, the derived formulae provide a common framework for obtaining related identities under appropriate parameter choices. The results are theoretical and are formulated under the parameter and convergence conditions stated for the respective operators and series. Thus, the study organises fractional integration and differentiation formulae for the specified product within a unified generalized Wright hypergeometric representation.

Keywords: Generalized fractional calculus operators; fractional integration; fractional differentiation; extended Mittag-Leffler type function; generalized Wright hypergeometric function; Srivastava polynomials; Saigo fractional calculus operators; Riemann-Liouville operators; Appell function; special functions.

Mathematics subject classification: 26A33, 33B15, 33C45, 33C70, 33E12.

1 Introduction

In 1903, the Swedish mathematician Gosta Mittag-Leffler [15] studied the function $E_{\mu}(z)$ and defined it as

$$E_{\mu}(z)=\sum_{r=0}^{\infty}\frac{z^{r}}{\Gamma(\mu r+1)},\left(\mu,z\in C,\Re(\mu)>0\right), \tag{1.1}$$

The Mittag-Leffler function (1.1) is a generalisation of $exp(z)$ for $\mu=1$.

A generalisation of the function $E_{\mu}(z)$ was introduced by Wiman [34] and expressed as

$$E_{\mu,\upsilon}(z)=\sum_{r=0}^{\infty}\frac{z^{r}}{\Gamma(\mu r+\upsilon)},\left(\mu,\upsilon,z\in C,\Re(\mu)>0,\Re(\upsilon)>0\right). \tag{1.2}$$

Further, in 1971, a new generalisation of the Mittag-Leffler function was studied by Prabhakar [20] in terms of the following series representation:

$$E_{\mu,\upsilon}^{\lambda}(z)=\sum_{r=0}^{\infty}\frac{(\lambda)_r z^r}{\Gamma(\mu r+\upsilon)r!},\ (\mu,\upsilon,\lambda,z\in C,\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>0) \tag{1.3}$$

where $(\lambda)_r=\frac{\Gamma(\lambda+r)}{\Gamma(\lambda)}$ denotes the Pochhammer symbol (see [5], [7]).

In 2015, Parmar [19] defined the extended Mittag-Leffler type function as follows (see also [2, 3, 27]):

$$\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(z;p)=\sum_{r=0}^{\infty}\frac{\mathrm{B}^{\{k_\ell\}_{\ell\in N_0}}(\lambda+r,1-\lambda;p)}{\mathrm{B}(\lambda,1-\lambda)}\times\frac{z^r}{\Gamma(\mu r+\upsilon)}, \tag{1.4}$$

where $z,\upsilon,\lambda\in C;\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>1;\ p\geq 0$ and $\mathrm{B}^{\left(\{k_\ell\}_{\ell\in N_0}\right)}(\alpha,\beta;p)$ is the extended beta function defined by Srivastava et al. [29] in the following integral form:

$$\mathrm{B}^{\left(\{k_\ell\}_{\ell\in N_0}\right)}(\alpha,\beta;p)=\int_0^1 z^{\alpha-1}(1-z)^{\beta-1}\times\Theta\left(\{k_\ell\}_{\ell\in N_0};\frac{-p}{z(1-z)}\right)dz, \tag{1.5}$$

where $min\{\Re(\alpha),\Re(\beta)\}>0;\ p\geq 0$ and $\Theta\left(\{k_\ell\}_{\ell\in N_0};y\right)$ is a function of an appropriately bounded sequence $\{k_\ell\}_{\ell\in N_0}$ of arbitrary numbers (real or complex) defined as (see [29], p. 243, Eq. (2.1))

$$\Theta\left(\{k_\ell\}_{\ell\in N_0};y\right)=\begin{cases}\sum_{\ell=0}^{\infty}k_\ell\frac{y^\ell}{\ell!} & (|y|<r;0<r<\infty;k_0=1)\\ \mathrm{M}_0\,y^\rho\exp(y)\left[1+O\left(\frac{1}{y}\right)\right] & (\Re(y)\to\infty,\mathrm{M}_0>0,\rho\in C)\end{cases}. \tag{1.6}$$

Here, 'r' denotes the radius of convergence for some suitable constants M_0 *and* ρ, depending on the sequence $\{k_\ell\}_{\ell\in N_0}$.

Remark 1: If we take $k_\ell=\frac{(\rho)_\ell}{(\sigma)_\ell}(\ell\in N_0)$ in (1.4), then $\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(z;p)$ reduces to another form of extended Mittag-Leffler function $\mathrm{E}_{\mu,\nu}^{(\rho,\sigma);\lambda}(z;p)$ (see [19], p-1072, eq. (17)) as (see also [2, 3, 27])

$$\mathrm{E}_{\mu,\nu}^{(\rho,\sigma);\lambda}(z;p)=\sum_{r=0}^{\infty}\frac{\mathrm{B}^{(\rho,\sigma)}(\lambda+r,1-\lambda;p)}{\mathrm{B}(\lambda,1-\lambda)}\times\frac{z^r}{\Gamma(\mu r+\upsilon)}, \tag{1.7}$$

$(z,\upsilon,\lambda\in C;\Re(\rho)>0,\Re(\sigma)>0,\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>1;\ p\geq 0).$

Remark 2: If we set $k_\ell=1(\ell\in\mathrm{N}_0)$ in (1.4), then $\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(z;p)$ reduces to

$\mathrm{E}_{\mu,\upsilon}^{\lambda}(z;p)$ given by Özarslan and Yilmaz [17] as (see also [2, 3, 27])

$$\mathrm{E}_{\mu,\upsilon}^{\lambda}(z;p)=\sum_{r=0}^{\infty}\frac{\mathrm{B}(\lambda+r,1-\lambda;p)}{\mathrm{B}(\lambda,1-\lambda)}\times\frac{z^{r}}{\Gamma(\mu r+\upsilon)}, \tag{1.8}$$

$$(z,\upsilon,\lambda\in C;\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>1;p\geq 0).$$

Remark 3: If we take $k_{\ell}=1(\ell\in N_0)\,and\; p=0$, in (1.4), then $\mathrm{E}_{\mu,\upsilon}^{(\{k_\ell\}_{\ell\in N_0};\lambda)}(z;p)$ reduces to the Prabhakar-type Mittag-Leffler function $E_{\mu,\upsilon}^{\lambda}(z)$ (see [6, 11, 25]).

Remark 4: If we put $\lambda=1$ and $\lambda=\upsilon=1$ in Prabhakar-type Mittag-Leffler function $\mathrm{E}_{\mu,\upsilon}^{\lambda}(z;p)$, then it reduces to Wiman-type Mittag-Leffler function $\mathrm{E}_{\mu,\upsilon}^{1}(z)=\mathrm{E}_{\mu,\upsilon}(z)$ and Gosta Mittag-Leffler function $\mathrm{E}_{\mu,1}^{1}(z)=\mathrm{E}_{\mu}(z)$ respectively (see [6, 11, 25]).

In 2018, Agarwal et al. [1] introduced a useful extension of the Wright-type hypergeometric function as follows:

$$\begin{aligned}{}_{m+1}\psi_{n+1}^{(\{k_\ell\}_{\ell\in N_0})}(\xi;p)&={}_{m+1}\psi_{n+1}^{(\{k_\ell\}_{\ell\in N_0})}\left[\begin{matrix}(a_i,\mathrm{A}_i)_{1,m},(\lambda,1)\\(b_j,\mathrm{B}_j)_{1,n},(d,1)\end{matrix};(\xi;p)\right]\\&=\frac{1}{\Gamma(d-\lambda)}\sum_{r=0}^{\infty}\frac{\prod_{i=1}^{m}\Gamma(a_i+\mathrm{A}_i r)}{\prod_{j=1}^{n}\Gamma(b_j+\mathrm{B}_j r)}\mathrm{B}^{(\{k_\ell\}_{\ell\in N_0})}(\lambda+r,d-\lambda;p)\times\frac{\xi^{r}}{r!},\end{aligned} \tag{1.9}$$

where $\xi,\lambda\in C,\Re(d)>\Re(\lambda);p\geq 0$.

In 2015, Sharma and Devi [26] extended the Wright-type hypergeometric function as follows:

$$\begin{aligned}{}_{m+1}\psi_{n+1}(\xi;p)&={}_{m+1}\psi_{n+1}\left[\begin{matrix}(a_i,\mathrm{A}_i)_{1,m},(\lambda,1)\\(b_j,\mathrm{B}_j)_{1,n},(d,1)\end{matrix};(\xi;p)\right]\\&=\frac{1}{\Gamma(d-\lambda)}\sum_{r=0}^{\infty}\frac{\prod_{i=1}^{m}\Gamma(a_i+\mathrm{A}_i r)}{\prod_{j=1}^{n}\Gamma(b_j+\mathrm{B}_j r)}\mathrm{B}_p(\lambda+r,d-\lambda)\times\frac{\xi^{r}}{r!},\end{aligned} \tag{1.10}$$

where $\xi,\lambda\in C,\Re(p)>0,\Re(d)>\Re(\lambda)>0$ and $B_p(x,y)$ is the extended beta function (see [26], p. 46, Eq. (3)).

The generalized Wright hypergeometric function was defined by Wright [35] in the following series:

$$
{}_m\psi_n(\xi) = {}_m\psi_n\left[\begin{matrix}(a_i, A_i)_{1,m} \\ (b_j, B_j)_{1,n}\end{matrix};\xi\right] = \sum_{r=0}^{\infty}\frac{\prod_{i=1}^{m}\Gamma(a_i + A_i r)}{\prod_{j=1}^{n}\Gamma(b_j + B_j r)}\frac{\xi^r}{r!}, \tag{1.11}
$$

where $\xi, a_i, b_j \in C$ and $A_i, B_j \in R - \{0\}, (i = 1, \ldots, m; j = 1, \ldots, n)$ with condition $\sum_{j=1}^{q} B_j - \sum_{i=1}^{p} A_i > -1.$

Remark 5: If we take $k_\ell = 1(\ell \in N_0)$, then (1.9) reduces to (1.10); further, if we put $p = 0$, then (1.9) reduces to (1.11). (see [2, 3])

The general class of polynomials (also known as Srivastava polynomials) $S_n^m[\xi]$ is defined by Srivastava [30, 31] as follows: (see also [8, 12, 25])

$$
S_n^m[\xi] = \sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}\quad A_{n,s}\,\xi^s, \qquad (n = 0,1,2,...), \tag{1.12}
$$

where m is an arbitrary positive integer and the coefficients $A_{n,s}(n, s \geq 0)$ are arbitrary constants, real or complex. On suitably specialising the coefficients $A_{n,s}$, the polynomial family $S_n^m[\xi]$ yields a number of known polynomials as its special cases. These include, among others, the Hermite polynomials, the Jacobi polynomials, the Laguerre polynomials, the Bessel polynomials and several others (see [32, 33]).

2 Generalized fractional calculus operators

For $\alpha, \alpha', \eta, \eta', \delta \in C, x > 0$ and $\Re(\delta) > 0$, the generalized fractional integral and differential operators involving Appell's function (also known as Horn's function) $F_3(.)$ are defined by Saigo and Maeda (see [22], p. 393, Eq. (4.12) and (4.13)) as follows: (see also [12, 14, 23, 24, 25])

$$
\left(I_{0+}^{\alpha,\alpha',\eta,\eta',\delta}f\right)(x) = \frac{x^{-\alpha}}{\Gamma(\delta)}\int_0^x (x-t)^{\delta-1}t^{-\alpha'}F_3\left(\alpha,\alpha',\eta,\eta';\delta;1-\frac{t}{x},1-\frac{x}{t}\right)f(t)\,dt, \tag{2.1}
$$

$$
\left(I_{-}^{\alpha,\alpha',\eta,\eta',\delta}f\right)(x) = \frac{x^{-\alpha'}}{\Gamma(\delta)}\int_x^\infty (t-x)^{\delta-1}t^{-\alpha}F_3\left(\alpha,\alpha',\eta,\eta';\delta;1-\frac{x}{t},1-\frac{t}{x}\right)f(t)\,dt, \tag{2.2}
$$

$$
\left(D_{0+}^{\alpha,\alpha',\eta,\eta',\delta}f\right)(x) = \left(I_{0+}^{-\alpha',-\alpha,-\eta',-\eta,-\delta}f\right)(x) \tag{2.3}
$$

$$
= \left(\frac{d}{dx}\right)^k\left(I_{0+}^{-\alpha',-\alpha,-\eta'+k,-\eta,-\delta+k}f\right)(x), \left(\Re(\delta) > 0; k = [\Re(\delta)+1]\right) \tag{2.4}
$$

and

$$\left(D_{-}^{\alpha,\alpha',\eta,\eta',\delta}f\right)(x)=\left(I_{-}^{-\alpha',-\alpha,-\eta',-\eta,-\delta}f\right)(x) \tag{2.5}$$

$$=\left(\frac{-d}{dx}\right)^{k}\left(I_{-}^{-\alpha',-\alpha,-\eta',-\eta+k,-\delta+k}f\right)(x),\left(\Re(\delta)>0;k=\left[\Re(\delta)+1\right]\right) \tag{2.6}$$

For $\alpha,\eta,\delta\in C$ and $x>0$, the Saigo fractional calculus operators [21] associated with the Gauss Gauss hypergeometric function ${}_2F_1(.)$ are defined for $\Re(\alpha)>0$, as follows:

$$\left(I_{0+}^{\alpha,\eta,\delta}f\right)(x)=\frac{x^{-\alpha-\eta}}{\Gamma(\delta)}\int_0^x (x-t)^{\alpha-1}\ {}_2F_1\left(\alpha+\eta,-\delta;\alpha;1-\frac{t}{x}\right)f(t)\,dt, \tag{2.7}$$

$$\left(I_{-}^{\alpha,\eta,\delta}f\right)(x)=\frac{1}{\Gamma(\delta)}\int_x^\infty (t-x)^{\alpha-1}\ t^{-\alpha-\eta}\ {}_2F_1\left(\alpha+\eta,-\delta;\alpha;1-\frac{x}{t}\right)f(t)\,dt, \tag{2.8}$$

$$\left(D_{0+}^{\alpha,\eta,\delta}f\right)(x)=\left(I_{0+}^{-\alpha,-\eta,\alpha+\delta}f\right)(x)=\left(\frac{d}{dx}\right)^{k}\left(I_{0+}^{-\alpha+k,-\eta-k,\alpha+\delta-k}f\right)(x),(k=\left[\Re(\alpha)+1\right]), \tag{2.9}$$

and

$$\left(D_{-}^{\alpha,\eta,\delta}f\right)(x)=\left(I_{-}^{-\alpha,-\eta,\alpha+\delta}f\right)(x)=\left(\frac{-d}{dx}\right)^{k}\left(I_{-}^{-\alpha+k,-\eta-k,\alpha+\delta}f\right)(x),(k=\left[\Re(\alpha)+1\right]). \tag{2.10}$$

Remark 6: If we take $\alpha=\alpha+\eta,\alpha'=\eta'=0,\eta=-\delta$ and $\delta=\alpha$, operators given in (2.1)-(2.6) reduce to the Saigo fractional calculus operators as given by (2.7)-(2.10). (see [4, 9, 10])
Remark 7: If we set $\eta=-\alpha$, then operators (2.7)-(2.10) reduce to Riemann-Liouville fractional calculus operators [,] as follows: (see [4, 9, 10])

$$\left(I_{0+}^{\alpha,-\alpha,\delta}f\right)(x)=\left(I_{0+}^{\alpha}f\right)(x), \tag{2.11}$$

$$\left(I_{-}^{\alpha,-\alpha,\delta}f\right)(x)=\left(I_{-}^{\alpha}f\right)(x), \tag{2.12}$$

$$\left(D_{0+}^{\alpha,-\alpha,\delta}f\right)(x)=\left(D_{0+}^{\alpha}f\right)(x), \tag{2.13}$$

and

$$\left(D_{-}^{\alpha,-\alpha,\delta}f\right)(x)=\left(D_{-}^{\alpha}f\right)(x), \tag{2.14}$$

Further, the following image formulas for a power function under operators (2.1) and (2.2) are given by (see [5, 6, 24, 25])

$$\left(I_{0+}^{\alpha,\alpha',\eta,\eta',\delta}t^{\sigma-1}\right)(x)=x^{\sigma-\alpha-\alpha'+\delta-1}\,\Gamma\left[\begin{matrix}\sigma,\sigma+\delta-\alpha-\alpha'-\eta,\sigma+\eta'-\alpha'\\ \sigma+\delta-\alpha-\alpha',\sigma+\delta-\alpha'-\eta,\rho+\eta'\end{matrix}\right], \tag{2.15}$$

where $\Re(\delta)>0,\Re(\sigma)>max\left[0,\Re(\alpha+\alpha'+\eta-\delta),\Re(\alpha'-\eta')\right]$,

and

$$\left(I_{-}^{\alpha,\alpha',\eta,\eta',\delta}t^{\sigma-1}\right)(x)=x^{\sigma-\alpha-\alpha'+\delta-1}\,\Gamma\left[\begin{matrix}1+\alpha+\alpha'-\delta-\sigma,1+\alpha+\eta'-\delta-\sigma,1-\eta-\sigma\\ 1-\sigma,1+\alpha+\alpha'+\eta'-\delta-\sigma,1+\alpha-\eta-\sigma\end{matrix}\right], \tag{2.16}$$

where $\Re(\delta)>0, \Re(\sigma)<1+\min\left[\Re(-\eta), \Re(\alpha+\alpha'-\delta), \Re(\alpha+\eta'-\delta)\right]$.

Here, the Symbol $\Gamma\left[\begin{matrix}\alpha,\beta,\gamma\\ \mu,\upsilon,\lambda\end{matrix}\right]$ will be used to represent the ratio of product of gamma functions as $\dfrac{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma)}{\Gamma(\mu)\Gamma(\upsilon)\Gamma(\lambda)}$.

Research Gap: Related studies have examined fractional differential equations, Mittag-Leffler-type functions, Srivastava polynomials, Saigo fractional calculus operators, Riemann-Liouville-type operators, and generalized fractional-operator frameworks (Gupta et al. [5-7]; Shaktawat et al. [25]; Kumar et al. [10]; Nishant et al. [16]; Pandey et al. [18]; Singh et al. [28]; Kumar [13]; Sharma et al. [27]; Bhadana et al. [2, 3]). However, the image formulae for fractional integral and differential operators acting on the product of a general class of polynomials (Srivastava polynomials) and Parmar's extended Mittag-Leffler-type function, expressed through the generalized Wright hypergeometric function, have not yet been consolidated in the existing literature.

3 Main results

3.1 Generalized fractional integration of product of extended Mittag-Leffler function and Srivastava polynomials

In this section, we establish two image formulae for left-sided and right-sided generalized fractional integration involving the product of the general class of polynomials (also known as Srivastava polynomials) and an extended Mittag-Leffler type function in terms of the generalized Wright hypergeometric function. These formulae are stated in the following theorems:

Theorem 1 Let $\alpha,\alpha',\eta,\eta',\delta,\mu,\upsilon,\lambda,p\in C, \Re(\mu)>0, \Re(\upsilon)>0, \Re(\lambda)>1, p\geq 0, t,x>0$ and $a,\omega\in R$ such that $\Re(\delta)>0$ and $\Re(\upsilon)>max\{0,\Re(\alpha+\alpha'+\eta-\delta),\Re(\alpha'-\eta')\}$. Then the left-sided generalized fractional integration $I_{0+}^{\alpha,\alpha',\eta,\eta',\delta}$ of the product of the general class of polynomials $S_n^m[z]$ and extended Mittag-Leffler type function $\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(z;p)$ and is given by

$$\{I_{0+}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\upsilon-1}S_n^m\left[\omega t^\beta\right]\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(at;p)\right)\}(x)=\frac{x^{\upsilon+\delta-\alpha-\alpha'-1}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^\beta\right)^s$$

$$\times{}_4\psi_4^{\left(\{k_\ell\}_{\ell\in N_0}\right)}\left[\begin{matrix}(\lambda,1),(\upsilon+\beta s,1),(\upsilon+\beta s+\delta-\alpha-\alpha'-\eta,1),(\upsilon+\beta s+\eta'-\alpha',1)\\ (\upsilon,\mu),(\upsilon+\beta s+\delta-\alpha-\alpha',1),(\upsilon+\beta s+\delta-\alpha'-\eta,1),(\upsilon+\beta s+\eta',1)\end{matrix};(ax;p)\right] \quad (3.1)$$

Proof: Using the series representation of extended Mittag-Leffler type function (1.4), the general class of polynomials (1.12) and left-sided fractional integration power function formula (2.15), the LHS of theorem leads to

$$\{I_{0+}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\upsilon-1}S_n^m\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(at;p)\right)\}(x)$$

$$=\left\{I_{0+}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\upsilon-1}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega t^{\beta}\right)^s\times\sum_{r=0}^{\infty}\frac{\mathrm{B}^{\left(\{k_\ell\}_{\ell\in N_0}\right)}(\lambda+r,1-\lambda;p)}{\mathrm{B}(\lambda,1-\lambda)}\times\frac{(at)^r}{\Gamma(\mu r+\upsilon)}\right)\right\}(x),$$

By interchanging the order of integration and summations, which is valid under the conditions of Theorem 1, we get

$$=\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\,\omega^s\times\sum_{r=0}^{\infty}\frac{\mathrm{B}^{\left(\{k_\ell\}_{\ell\in N_0}\right)}(\lambda+r,1-\lambda;p)}{\mathrm{B}(\lambda,1-\lambda)}\times\frac{a^r}{\Gamma(\mu r+\upsilon)}\left(I_{0+}^{\alpha,\alpha',\eta,\eta',\delta}t^{\upsilon+\beta s+r-1}\right)(x)$$

$$=\frac{x^{\upsilon+\delta-\alpha-\alpha'-1}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^s\times\sum_{r=0}^{\infty}\frac{\mathrm{B}^{\left(\{k_\ell\}_{\ell\in N_0}\right)}(\lambda+r,1-\lambda;p)}{\mathrm{B}(\lambda,1-\lambda)}$$

$$\times\frac{\Gamma(\upsilon+\beta s+r)\Gamma(\upsilon+\beta s+\delta-\alpha-\alpha'-\eta+r)\Gamma(\upsilon+\beta s+\eta'-\alpha'+r)}{\Gamma(\upsilon+\beta s+\delta-\alpha-\alpha'+r)\Gamma(\upsilon+\beta s+\delta-\alpha'-\eta+r)\Gamma(\upsilon+\beta s+\eta'+r)}\times\frac{(ax)^r}{\Gamma(\mu r+\upsilon)},\qquad(3.2)$$

Next, by interpreting the terms of the above equation (3.2), in view of definition (1.9), we arrive at the result (3.1), which completes the proof of Theorem 1.

On setting $n=0, A_{0,0}=1$ then $S_0^m[z]\to 1$ in Theorem 1, we obtain the following result:

Corollary 1.1 Let $\alpha,\alpha',\eta,\eta',\delta,\mu,\upsilon,\lambda,p\in C,\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>1,p\geq 0,t,x>0$ and $a,\omega\in R$ such that $\Re(\delta)>0$ and $\Re(\upsilon)>max\{0,\Re(\alpha+\alpha'+\eta-\delta),\Re(\alpha'-\eta')\}$. Then we have the formula:

$$\{I_{0+}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\upsilon-1}\,\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(at;p)\right)\}(x)$$

$$=\frac{x^{\upsilon+\delta-\alpha-\alpha'-1}}{\Gamma(\lambda)}\times{}_4\psi_4^{\left(\{k_\ell\}_{\ell\in N_0}\right)}\left[\begin{matrix}(\lambda,1),(\upsilon,1),(\upsilon+\delta-\alpha-\alpha'-\eta,1),(\upsilon+\eta'-\alpha',1)\\(\upsilon,\mu),(\upsilon+\delta-\alpha-\alpha',1),(\upsilon+\delta-\alpha'-\eta,1),(\upsilon+\eta',1)\end{matrix};(ax;p)\right]\qquad(3.3)$$

On taking $\alpha=\alpha+\eta,\alpha'=\eta'=0,\eta=-\delta$ and $\delta=\alpha$, Theorem 1 reduces to the following corollary:

Corollary 1.2 Let $\alpha,\beta,\eta,\mu,\upsilon,\lambda,p\in C,\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>1,p\geq 0,t,x>0$ and $a,\omega\in R$ such that $\Re(\alpha)>0$ and $\Re(\upsilon)>max\{0,\Re(\eta-\delta)\}$. Then the following integral formula holds:

$$\{I_{0+}^{\alpha,\eta,\delta}\left(t^{\upsilon-1}S_n^m\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(at;p)\right)\}(x)=\frac{x^{\upsilon-\eta-1}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^s$$

$$\times {}_3\psi_3^{\left(\{k_\ell\}_{\ell\in N_0}\right)}\left[\begin{matrix}(\lambda,1),(\upsilon+\beta s,1),(\upsilon+\beta s+\delta-\eta,1)\\(\upsilon,\mu),(\upsilon+\beta s-\eta,1),(\upsilon+\beta s+\delta+\alpha,1)\end{matrix};(ax;p)\right] \tag{3.4}$$

Now, on putting $\eta=-\alpha$ in Corollary 1.2, we get the following result:

Corollary 1.3 Let $\alpha,\mu,\upsilon,\lambda,p\in C,\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>1,p\geq 0,t,x>0$ and $a,\ \omega\in R$ such that $\Re(\alpha)>0$ Then we have the following formula:

$$\{I_{0+}^{\alpha}\left(t^{\upsilon-1}S_n^m\left[\omega t^\beta\right]\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(at;p)\right)\}(x)$$

$$=\frac{x^{\upsilon+\alpha-1}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^\beta\right)^s\times {}_2\psi_2^{\left(\{k_\ell\}_{\ell\in N_0}\right)}\left[\begin{matrix}(\lambda,1),(\upsilon+\beta s,1)\\(\upsilon,\mu),(\upsilon+\beta s+\alpha,1)\end{matrix};(ax;p)\right] \tag{3.5}$$

Next, if we put $k_\ell=\frac{(\rho)_\ell}{(\sigma)}(\ell\in N_0)$ in (3.1), then $\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(z;p)$ reduces to $\mathrm{E}_{\mu,\upsilon}^{(\rho,\sigma);\lambda}(z;p)$, we arrive at the following result:

Corollary 1.4 Let $\alpha,\alpha',\eta,\eta',\delta,\rho,\sigma,\mu,\upsilon,\lambda,p\in C,\Re(\rho)>0,\Re(\sigma)>0,\Re(\mu)>0,\Re(\upsilon)>0,$ $\Re(\lambda)>1,p\geq 0,t,x>0$ and $a,\ \omega\in R$ such that $\Re(\delta)>0$ and $\Re(\upsilon)>max\{0,\Re(\alpha+\alpha'+\eta-\delta),\Re(\alpha'-\eta')\}$. Then we have the following formula:

$$\{I_{0+}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\upsilon-1}S_n^m\left[\omega t^\beta\right]\mathrm{E}_{\mu,\upsilon}^{(\rho,\sigma);\lambda}(at;p)\right)\}(x)=\frac{x^{\upsilon+\delta-\alpha-\alpha'-1}\Gamma(\sigma)}{\Gamma(\lambda)\Gamma(\rho)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^\beta\right)^s$$

$$\times {}_5\psi_5\left[\begin{matrix}(\lambda,1),(\upsilon+\beta s,1),(\upsilon+\beta s+\delta-\alpha-\alpha'-\eta,1),(\upsilon+\beta s+\eta'-\alpha',1),(\rho,1)\\(\upsilon,\mu),(\upsilon+\beta s+\delta-\alpha-\alpha',1),(\upsilon+\beta s+\delta-\alpha'-\eta,1),(\upsilon+\beta s+\eta',1),(\sigma,1)\end{matrix};(ax;p)\right] \tag{3.6}$$

Again, if we set $k_\ell=1(\ell\in N_0)$ in (3.1), then $\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(z;p)$ reduces to $\mathrm{E}_{\mu,\nu}^{\lambda}(z;p)$, we get the following corollary:

Corollary 1.5 Let $\alpha,\alpha',\eta,\eta',\delta,\mu,\upsilon,\lambda,p\in C,\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>1,p\geq 0,t,x>0$ and $a,\ \omega\in R$ such that $\Re(\delta)>0$ and $\Re(\upsilon)>max\{0,\Re(\alpha+\alpha'+\eta-\delta),\Re(\alpha'-\eta')\}$. Then the following result holds:

$$\{I_{0+}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\upsilon-1}S_n^m\left[\omega t^\beta\right]\mathrm{E}_{\mu,\upsilon}^{\lambda}(at;p)\right)\}(x)=\frac{x^{\upsilon+\delta-\alpha-\alpha'-1}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^\beta\right)^s$$

$$\times {}_4\psi_4\left[\begin{matrix}(\lambda,1),(\upsilon+\beta s,1),(\upsilon+\beta s+\delta-\alpha-\alpha'-\eta,1),(\upsilon+\beta s+\eta'-\alpha',1)\\(\upsilon,\mu),(\upsilon+\beta s+\delta-\alpha-\alpha',1),(\upsilon+\beta s+\delta-\alpha'-\eta,1),(\upsilon+\beta s+\eta',1)\end{matrix};(ax;p)\right] \quad (3.7)$$

Further, if we take $k_\ell = 1(\ell \in N_0)$ *and* $p=0$ in (3.1), then $\mathrm{E}_{\mu,\upsilon}^{(\{k_\ell\}_{\ell\in N_0};\lambda)}(z;p)$ reduce to $\mathrm{E}_{\mu,\upsilon}^{\lambda}(z)$, then we obtain the following result:

Corollary 1.6 Let $\alpha,\alpha',\eta,\eta',\delta,\mu,\upsilon,\lambda \in C, \Re(\mu)>0, \Re(\upsilon)>0, \Re(\lambda)>1, t,x>0$ and $a,\,\omega \in R$ such that $\Re(\delta)>0$ and $\Re(\upsilon) > max\{0, \Re(\alpha+\alpha'+\eta-\delta), \Re(\alpha'-\eta')\}$. Then we have the following formula:

$$\{I_{0+}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\upsilon-1}S_n^m\left[\omega t^\beta\right]\mathrm{E}_{\mu,\upsilon}^{\lambda}(at)\right)\}(x) = \frac{x^{\upsilon+\delta-\alpha-\alpha'-1}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^\beta\right)^s$$

$$\times {}_4\psi_4\left[\begin{matrix}(\lambda,1),(\upsilon+\beta s,1),(\upsilon+\beta s+\delta-\alpha-\alpha'-\eta,1),(\upsilon+\beta s+\eta'-\alpha',1)\\(\upsilon,\mu),(\upsilon+\beta s+\delta-\alpha-\alpha',1),(\upsilon+\beta s+\delta-\alpha'-\eta,1),(\upsilon+\beta s+\eta',1)\end{matrix};ax\right] \quad (3.8)$$

Theorem 2 Let $\alpha,\alpha',\eta,\eta',\delta,\mu,\upsilon,\lambda,p \in C, \Re(\mu)>0, \Re(\upsilon)>0, \Re(\lambda)>1, p\geq 0, t,x>0$, and $a,\,\omega\in R$ such that $\Re(\delta)>0$ and $\Re(1-\delta-\upsilon) < 1+min\{\Re(-\eta), \Re(\alpha+\alpha'-\delta), \Re(\alpha+\eta'-\delta)\}$. Then the right-sided generalized fractional integration $I_{-}^{\alpha,\alpha',\eta,\eta',\delta}$ of the product of the general class of polynomials $S_n^m[z]$ and extended Mittag-Leffler type function $\mathrm{E}_{\mu,\upsilon}^{(\{k_\ell\}_{\ell\in N_0};\lambda)}(z;p)$ is given by

$$\{I_{-}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{-\upsilon-\delta}S_n^m\left[\omega t^\beta\right]\mathrm{E}_{\mu,\upsilon}^{(\{k_\ell\}_{\ell\in N_0};\lambda)}\left(at^{-1};p\right)\right)\}(x) = \frac{x^{-\upsilon-\alpha-\alpha'}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^\beta\right)^s$$

$$\times {}_4\psi_4^{(\{k_\ell\}_{\ell\in N_0})}\left[\begin{matrix}(\lambda,1),(\upsilon-\beta s+\alpha+\alpha',1),(\upsilon-\beta s+\alpha+\eta',1),(\upsilon-\beta s+\delta-\eta,1)\\(\upsilon,\mu),(\upsilon-\beta s+\delta,1),(\upsilon-\beta s+\alpha+\alpha'+\eta',1),(\upsilon-\beta s+\delta+\alpha-\eta,1)\end{matrix};(ax^{-1};p)\right] \quad (3.9)$$

Proof: Using the series representation of the extended Mittag-Leffler function (1.4), the general class of polynomials (1.12), and the right-sided fractional-integration power-function formula (2.16), the left-hand side of Theorem 2 can be written as

$$\{I_{-}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{-\upsilon-\delta}S_n^m\left[\omega t^\beta\right]\mathrm{E}_{\mu,\upsilon}^{(\{k_\ell\}_{\ell\in N_0};\lambda}\left(at^{-1};p\right)\right)\}(x)$$

$$=\left\{I_{-}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{-\upsilon-\delta}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega t^\beta\right)^s \times \sum_{r=0}^{\infty}\frac{\mathrm{B}^{(\{k_\ell\}_{\ell\in N_0})}(\lambda+r,1-\lambda;p)}{\mathrm{B}(\lambda,1-\lambda)}\times\frac{\left(at^{-1}\right)^r}{\Gamma(\mu r+\upsilon)}\right)\right\}(x),$$

By interchanging the order of integration and summations, which is valid under the conditions of Theorem 2, we have

$$= \sum_{s=0}^{[n/m]} \frac{(-n)_{ms}}{s!} A_{n,s}\, \omega^s \times \sum_{r=0}^{\infty} \frac{\mathrm{B}^{\left(\{k_\ell\}_{\ell\in N_0}\right)}(\lambda+r,1-\lambda;p)}{\mathrm{B}(\lambda,1-\lambda)} \times \frac{a^r}{\Gamma(\mu r+\upsilon)} \left(I_{-}^{\alpha,\alpha',\eta,\eta',\delta} t^{1-\upsilon-\delta+\beta s-r-1}\right)(x)$$

$$= \frac{x^{-\upsilon-\alpha-\alpha'}}{\Gamma(\lambda)} \sum_{s=0}^{[n/m]} \frac{(-n)_{ms}}{s!} A_{n,s} \left(\omega x^{\beta}\right)^s \times \sum_{r=0}^{\infty} \frac{\mathrm{B}^{\left(\{k_\ell\}_{\ell\in N_0}\right)}(\lambda+r,1-\lambda;p)}{\mathrm{B}(\lambda,1-\lambda)}$$

$$\times \frac{\Gamma(\upsilon-\beta s+\alpha+\alpha'+r)\Gamma(\upsilon-\beta s+\alpha+\eta'+r)\Gamma(\upsilon-\beta s+\delta-\eta+r)}{\Gamma(\upsilon-\beta s+\delta+r)\Gamma(\upsilon-\beta s+\alpha+\alpha'+\eta'+r)\Gamma(\upsilon-\beta s+\delta+\alpha-\eta+r)} \times \frac{\left(ax^{-1}\right)^r}{\Gamma(\mu r+\upsilon)}, \quad (3.10)$$

Now, interpreting the terms of the above equation (3.10), in view of definition (1.9), we arrive at the result (3.9), which completes the proof of Theorem 2.

On setting $n=0, A_{0,0}=1$ then $S_0^m[z]\to 1$ in (3.9), we obtain the following corollary:

Corollary 2.1 Let $\alpha,\alpha',\eta,\eta',\delta,\mu,\upsilon,\lambda,p\in C, \Re(\mu)>0, \Re(\upsilon)>0, \Re(\lambda)>1, p\geq 0, t,x>0,$ and $a,\omega\in R$ such that $\Re(\delta)>0$ and $\Re(1-\delta-\upsilon)<1+min\{\Re(-\eta),\Re(\alpha+\alpha'-\delta),\Re(\alpha+\eta'-\delta)\}$. Then we have the formula:

$$\{I_{-}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{-\upsilon-\delta}\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}\left(at^{-1};p\right)\right)\}(x)$$

$$= \frac{x^{-\upsilon-\alpha-\alpha'}}{\Gamma(\lambda)} \times {}_4\psi_4^{\left(\{k_\ell\}_{\ell\in N_0}\right)}\left[\begin{matrix}(\lambda,1),(\upsilon+\alpha+\alpha',1),(\upsilon+\alpha+\eta',1),(\upsilon+\delta-\eta,1)\\(\upsilon,\mu),(\upsilon+\delta,1),(\upsilon+\alpha+\alpha'+\eta',1),(\upsilon+\delta+\alpha-\eta,1)\end{matrix};\left(ax^{-1};p\right)\right] \quad (3.11)$$

If we take $\alpha=\alpha+\eta, \alpha'=\eta'=0, \eta=-\delta$ and $\delta=\alpha$, Theorem 2 reduces to the following corollary:

Corollary 2.2 Let $\alpha,\eta,\delta,\mu,\upsilon,\lambda,p\in C, \Re(\mu)>0, \Re(\upsilon)>0, \Re(\lambda)>1, p\geq 0, t,x>0$ and $a,\omega\in R$ such that $\Re(\alpha)>0$ and $\Re(1-\delta-\upsilon)<1+min\{\Re(-\eta),\Re(-\delta)\}$. Then we have

$$\{I_{-}^{\alpha,\eta,\delta}\left(t^{-\upsilon-\alpha}S_n^m\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}\left(at^{-1};p\right)\right)\}(x) = \frac{x^{-\upsilon-\alpha-\eta}}{\Gamma(\lambda)} \sum_{s=0}^{[n/m]} \frac{(-n)_{ms}}{s!} A_{n,s}\left(\omega x^{\beta}\right)^s$$

$$\times {}_3\psi_3^{\left(\{k_\ell\}_{\ell\in N_0}\right)}\left[\begin{matrix}(\lambda,1),(\upsilon-\beta s+\alpha+\eta,1),(\upsilon-\beta s+\delta+\alpha,1)\\(\upsilon,\mu),(\upsilon-\beta s+\alpha,1),(\upsilon-\beta s+\delta+2\alpha+\eta,1)\end{matrix};\left(ax^{-1};p\right)\right] \quad (3.12)$$

Further, if we put $\eta=-\alpha$ in Corollary 2.2, we arrive at the following result:

Corollary 2.3 Let $\alpha,\mu,\upsilon,\lambda,p\in C,\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>1,p\geq 0,x>0$ and $a,\ \omega\in R$ such that $\Re(\alpha)>0$. Then the following result holds:

$$\{I_{-}^{\alpha}\left(t^{-\upsilon-\alpha}S_n^m\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}\left(at^{-1};p\right)\right)\}(x)$$

$$=\frac{x^{-\upsilon}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^{s}\times{}_2\psi_2^{\left(\{k_\ell\}_{\ell\in N_0}\right)}\left[\begin{matrix}(\lambda,1),(\upsilon-\beta s,1)\\(\upsilon,\mu),(\upsilon-\beta s+\alpha,1)\end{matrix};\left(ax^{-1};p\right)\right] \tag{3.13}$$

Next, if we put $k_\ell=\frac{(\rho)_\ell}{(\sigma)}(\ell\in N_0)$ in (3.9), then $\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(z;p)$ reduce to $\mathrm{E}_{\mu,\upsilon}^{(\rho,\sigma);\lambda}(z;p)$, we arrive at the following result:

Corollary 2.4 Let $\alpha,\alpha',\eta,\eta',\delta,\rho,\sigma\mu,\upsilon,\lambda,p\in C,\Re(\rho)>0,\Re(\sigma)>0,\Re(\mu)>0,\Re(\upsilon)>0,$ $\Re(\lambda)>1,\ p\geq 0,\ t,x>0,$ and $a,\ \omega\in R$ such that $\Re(\delta)>0$ and $\Re(1-\delta-\upsilon)<1+min\{\Re(-\eta),\Re(\alpha+\alpha'-\delta),\Re(\alpha+\eta'-\delta)\}$. Then we have the following formula:

$$\{I_{-}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{-\upsilon-\delta}S_n^m\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{(\rho,\sigma);\lambda}\left(at^{-1};p\right)\right)\}(x)=\frac{x^{-\upsilon-\alpha-\alpha'}\Gamma(\sigma)}{\Gamma(\lambda)\Gamma(\rho)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^{s}$$

$$\times{}_5\psi_5\left[\begin{matrix}(\lambda,1),(\upsilon-\beta s+\alpha+\alpha',1),(\upsilon-\beta s+\alpha+\eta',1),(\upsilon-\beta s+\delta-\eta,1),(\rho,1)\\(\upsilon,\mu),(\upsilon-\beta s+\delta,1),(\upsilon-\beta s+\alpha+\alpha'+\eta',1),(\upsilon-\beta s+\delta+\alpha-\eta,1),(\sigma,1)\end{matrix};\left(ax^{-1};p\right)\right] \tag{3.14}$$

Again, if we set $k_\ell=1(\ell\in N_0)$ in (3.9), then $\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(z;p)$ reduces to $\mathrm{E}_{\mu,\upsilon}^{\lambda}(z;p)$, we obtain the following result:

Corollary 2.5 Let $\alpha,\alpha',\eta,\eta',\delta,\mu,\upsilon,\lambda,p\in C,\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>1,p\geq 0,t,x>0,$ and $a,\omega\in R$ such that $\Re(\delta)>0$ and $\Re(1-\delta-\nu)<1+min\{\Re(-\eta),\Re(\alpha+\alpha'-\delta),\Re(\alpha+\eta'-\delta)\}$. Then the following formula holds:

$$\{I_{-}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{-\upsilon-\delta}S_n^m\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{\lambda}\left(at^{-1};p\right)\right)\}(x)=\frac{x^{-\upsilon-\alpha-\alpha'}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^{s}$$

$$\times{}_4\psi_4\left[\begin{matrix}(\lambda,1),(\upsilon-\beta s+\alpha+\alpha',1),(\upsilon-\beta s+\alpha+\eta',1),(\upsilon-\beta s+\delta-\eta,1)\\(\upsilon,\mu),(\upsilon-\beta s+\delta,1),(\upsilon-\beta s+\alpha+\alpha'+\eta',1),(\upsilon-\beta s+\delta+\alpha-\eta,1)\end{matrix};\left(ax^{-1};p\right)\right] \tag{3.15}$$

Further, if we take $k_\ell=1(\ell\in N_0)$ and $p=0$ in (3.9), then $\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(z;p)$ reduces to $\mathrm{E}_{\mu,\upsilon}^{\lambda}(z)$, then we obtain the following result:

Corollary 2.6 Let $\alpha,\alpha',\eta,\eta',\delta,\mu,\upsilon,\lambda\in C,\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>1,t,x>0,$ and $a,\omega\in R$ such that $\Re(\delta)>0$ and $\Re(1-\delta-\upsilon)<1+min\{\Re(-\eta),\Re(\alpha+\alpha'-\delta),\Re(\alpha+\eta'-\delta)\}$. Then we have holds the following formula:

$$\{I_{-}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{-\upsilon-\delta}S_n^m\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{\lambda}\left(at^{-1}\right)\right)\}(x)=\frac{x^{-\upsilon-\alpha-\alpha'}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^{s}$$

$$\times{}_4\psi_4\left[\begin{matrix}(\lambda,1),(\upsilon-\beta s+\alpha+\alpha',1),(\upsilon-\beta s+\alpha+\eta',1),(\upsilon-\beta s+\delta-\eta,1)\\(\upsilon,\mu),(\upsilon-\beta s+\delta,1),(\upsilon-\beta s+\alpha+\alpha'+\eta',1),(\upsilon-\beta s+\delta+\alpha-\eta,1)\end{matrix};ax^{-1}\right]\quad(3.16)$$

3.2 Left-sided and right-sided generalized fractional differentiation of the product of an extended Mittag-Leffler type function and Srivastava polynomials

In this section, we establish two image formulae for left-sided and right-sided generalized fractional differentiation involving the product of the general class of polynomials (also known as Srivastava polynomials) and an extended Mittag-Leffler type function in terms of the generalized Wright hypergeometric function. These formulae are stated in the following theorems:

$\alpha,\alpha',\eta,\eta\ ,\delta,\mu,\upsilon,\lambda,p\in C,\Re\ \ \mu\ \ >0,\Re\ \ \upsilon\ \ >0,\Re\ \ \lambda\ \ >1,p\geq 0,t,x>0,$ $a,\omega\in R$ such that $\Re(\delta)>0$ and $\Re(\upsilon)>max\{0,\Re(\delta-\alpha-\alpha'-\eta'),\Re(\eta-\alpha)\}$. Then the left-sided generalized fractional differentiation $D_{0+}^{\alpha,\alpha',\eta,\eta',\delta}$ of the product of the general class of polynomials $S_n^m[z]$ and extended Mittag-Leffler type function $\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(z;p)$ is given by

$$\{D_{0+}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\upsilon-1}S_n^m\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(at;p)\right)\}(x)=\frac{x^{\upsilon-\delta+\alpha+\alpha'-1}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^{s}$$

$$\times{}_4\psi_4^{\left(\{k_\ell\}_{\ell\in N_0}\right)}\left[\begin{matrix}(\lambda,1),(\upsilon+\beta s,1),(\upsilon+\beta s-\delta+\alpha+\alpha'+\eta',1),(\upsilon+\beta s+\alpha-\eta,1)\\(\upsilon,\mu),(\upsilon+\beta s-\delta+\alpha+\alpha',1),(\upsilon+\beta s-\delta+\alpha+\eta',1),(\upsilon+\beta s-\eta,1)\end{matrix};(ax;p)\right]\quad(3.17)$$

Proof: Using (1.4) and (1.12), writing the function in series form, and applying (2.3) and (2.15), the left-hand side of Theorem 3 becomes

$$\{D_{0+}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\upsilon-1}S_n^m\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{\{k_\ell\}_{\ell\in N_0};\lambda}(at;p)\right)\}(x)$$

$$=\left\{D_{0+}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\upsilon-1}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega t^{\beta}\right)^{s}\times\sum_{r=0}^{\infty}\frac{\mathrm{B}^{\left(\{k_\ell\}_{\ell\in N_0}\right)}(\lambda+r,1-\lambda;p)}{\mathrm{B}(\lambda,1-\lambda)}\times\frac{(at)^{r}}{\Gamma(\mu r+\upsilon)}\right)\right\}(x),$$

By interchanging the order of differentiation and summations, which is valid under the conditions of Theorem 3, we have

$$=\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\ \omega^{s}\times\sum_{r=0}^{\infty}\frac{\mathrm{B}^{(\{k_\ell\}_{\ell\in N_0})}(\lambda+r,1-\lambda;p)}{\mathrm{B}(\lambda,1-\lambda)}\times\frac{a^{r}}{\Gamma(\mu r+\upsilon)}\left(D_{0+}^{\alpha,\alpha',\eta,\eta',\delta}t^{\upsilon+\beta s+r-1}\right)(x)$$

$$=\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\ \omega^{s}\times\sum_{r=0}^{\infty}\frac{\mathrm{B}^{(\{k_\ell\}_{\ell\in N_0})}(\lambda+r,1-\lambda;p)}{\mathrm{B}(\lambda,1-\lambda)}\times\frac{a^{r}}{\Gamma(\mu r+\upsilon)}\left(I_{0+}^{-\alpha',-\alpha,-\eta',-\eta,-\delta}t^{\upsilon+\beta s+r-1}\right)(x)$$

$$=\frac{x^{\upsilon-\delta+\alpha+\alpha'-1}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^{s}\times\sum_{r=0}^{\infty}\frac{\mathrm{B}^{(\{k_\ell\}_{\ell\in N_0})}(\lambda+r,1-\lambda;p)}{\mathrm{B}(\lambda,1-\lambda)}$$

$$\times\frac{\Gamma(\upsilon+\beta s+r)\Gamma(\upsilon+\beta s-\delta+\alpha+\alpha'+\eta'+r)\Gamma(\upsilon+\beta s+\alpha-\eta+r)}{\Gamma(\upsilon+\beta s-\delta+\alpha+\alpha'+r)\Gamma(\upsilon+\beta s-\delta+\alpha+\eta'+r)\Gamma(\upsilon+\beta s-\eta+r)}\times\frac{(ax)^{r}}{\Gamma(\mu r+\upsilon)},\tag{3.18}$$

Now, by interpreting the terms of the above equation (3.18), in view of definition (1.9), we arrive at the result (3.17), which completes the proof of Theorem 3.

On setting $n=0, A_{0,0}=1$ then $S_0^m[z]\to 1$ in (3.17), we obtain the following result:

Corollary 3.1 Let $\alpha,\alpha',\eta,\eta',\delta,\mu,\upsilon,\lambda,p\in C,\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>1,p\geq 0,t,x>0,$ and $a,\omega\in R$ such that $\Re(\delta)>0$ and $\Re(\upsilon)>max\{0,\Re(\delta-\alpha-\alpha'-\eta'),\Re(\eta-\alpha)\}$. Then we have the formula :

$$\{D_{0+}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\upsilon-1}\mathrm{E}_{\mu,\upsilon}^{(\{k_\ell\}_{\ell\in N_0};\lambda)}(at;p)\right)\}(x)$$

$$=\frac{x^{\upsilon-\delta+\alpha+\alpha'-1}}{\Gamma(\lambda)}\times{}_4\psi_4^{(\{k_\ell\}_{\ell\in N_0})}\left[\begin{matrix}(\lambda,1),(\upsilon,1),(\upsilon-\delta+\alpha+\alpha'+\eta',1),(\upsilon+\alpha-\eta,1)\\(\upsilon,\mu),(\upsilon-\delta+\alpha+\alpha',1),(\upsilon-\delta+\alpha+\eta',1),(\upsilon-\eta,1)\end{matrix};(ax;p)\right]\tag{3.19}$$

If we put $\alpha=\alpha+\eta,\alpha'=\eta'=0,\eta=-\delta$ and $\delta=\alpha$, Theorem 3 reduces to the following result:

Corollary 3.2 Let $\alpha,\eta,\delta,\mu,\upsilon,\lambda,p\in C,\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>1,p\geq 0,t,x>0$ and $a,\omega\in R$ such that $\Re(\alpha)>0$ and $\Re(\upsilon)>max\{0,\Re(\eta-\delta)\}$. Then the following integral formula holds:

$$\{D_{0+}^{\alpha,\eta,\delta}\left(t^{\upsilon-1}S_n^m\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{(\{k_\ell\}_{\ell\in N_0};\lambda)}(at;p)\right)\}(x)$$

$$=\frac{x^{\upsilon+\eta-1}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^{s}\times{}_3\psi_3^{(\{k_\ell\}_{\ell\in N_0})}\left[\begin{matrix}(\lambda,1),(\upsilon+\beta s,1),(\upsilon+\beta s+\delta+\alpha+\eta,1)\\(\upsilon,\mu),(\upsilon+\beta s+\eta,1),(\upsilon+\beta s+\delta,1)\end{matrix};(ax;p)\right]\tag{3.20}$$

Now, if we set $\eta=-\alpha$ in Corollary 3.2, we arrive at the following result:

Corollary 3.3 Let $\alpha,\mu,\upsilon,\lambda,p\in C,\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>1,p\geq 0,t,x>0$ and $a,\omega\in R$ such that $\Re(\alpha)>0$. Then the following result holds:

$$\{D_{0+}^{\alpha}\left(t^{\upsilon-1}S_n^m\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(at;p)\right)\}(x)$$

$$=\frac{x^{\upsilon-\alpha-1}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^{s}\times{}_2\psi_2^{\left(\{k_\ell\}_{\ell\in N_0}\right)}\left[\begin{matrix}(\lambda,1),(\upsilon+\beta s,1)\\(\upsilon,\mu),(\upsilon+\beta s-\alpha,1)\end{matrix};(ax;p)\right] \tag{3.21}$$

Next, if we put $k_\ell=\frac{(\rho)_\ell}{(\sigma)}(\ell\in N_0)$ in (3.17), then $\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(z;p)$ reduce to $\mathrm{E}_{\mu,\upsilon}^{(\rho,\sigma);\lambda}(z;p)$, we arrive at the following result:

Corollary 3.4 Let $\alpha,\alpha',\eta,\eta',\delta,\mu,\upsilon,\lambda,\rho,\sigma,p\in C,\Re(\rho)>0,\Re(\sigma)>0,\Re(\mu)>0,\Re(\upsilon)>0,$ $\Re(\lambda)>1,p\geq 0,t,x>0,$ and $a,\omega\in R$ such that $\Re(\delta)>0$ and $\Re(\upsilon)>max\{0,\Re(\delta-\alpha-\alpha'-\eta'),\Re(\eta-\alpha)\}$. Then the following formula holds:

$$\{D_{0+}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\upsilon-1}S_n^m\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{(\rho,\sigma);\lambda}(at;p)\right)\}(x)=\frac{x^{\upsilon-\delta+\alpha+\alpha'-1}\Gamma(\sigma)}{\Gamma(\lambda)\Gamma(\rho)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^{s}$$

$$\times{}_5\psi_5\left[\begin{matrix}(\lambda,1),(\upsilon+\beta s,1),(\upsilon+\beta s-\delta+\alpha+\alpha'+\eta',1),(\upsilon+\beta s+\alpha-\eta,1),(\rho,1)\\(\upsilon,\mu),(\upsilon+\beta s-\delta+\alpha+\alpha',1),(\upsilon+\beta s-\delta+\alpha+\eta',1),(\upsilon+\beta s-\eta,1),(\sigma,1)\end{matrix};(ax;p)\right] \tag{3.22}$$

Again, if we set $k_\ell=1(\ell\in N_0)$ in in (3.17), then $\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(z;p)$ reduce to $\mathrm{E}_{\mu,\upsilon}^{\lambda}(z;p)$, we get the following corollary:

Corollary 3.5 Let $\alpha,\alpha',\eta,\eta',\delta,\mu,\upsilon,\lambda,p\in C,\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>1,p\geq 0,t,x>0,$ and $a,\omega\in R$ such that $\Re(\delta)>0$ and $\Re(\upsilon)>max\{0,\Re(\delta-\alpha-\alpha'-\eta'),\Re(\eta-\alpha)\}$. Then the following result holds:

$$\{D_{0+}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\upsilon-1}S_n^m\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{\lambda}(at;p)\right)\}(x)=\frac{x^{\upsilon-\delta+\alpha+\alpha'-1}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^{s}$$

$$\times{}_4\psi_4\left[\begin{matrix}(\lambda,1),(\upsilon+\beta s,1),(\upsilon+\beta s-\delta+\alpha+\alpha'+\eta',1),(\upsilon+\beta s+\alpha-\eta,1)\\(\upsilon,\mu),(\upsilon+\beta s-\delta+\alpha+\alpha',1),(\upsilon+\beta s-\delta+\alpha+\eta',1),(\upsilon+\beta s-\eta,1)\end{matrix};(ax;p)\right] \tag{3.23}$$

Further, if we take $k_\ell=1(\ell\in N_0)\,and\;p=0$ in (3.1), then $\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(z;p)$ reduce to $\mathrm{E}_{\mu,\upsilon}^{\lambda}(z)$, then we obtain the following result:

Corollary 3.6 Let $\alpha,\alpha',\eta,\eta',\delta,\mu,\upsilon,\lambda\in C,\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>1,t,x>0,$ and $a,\ \omega\in R$ such that $\Re(\delta)>0$ and $\Re(\upsilon)>max\{0,\Re(\delta-\alpha-\alpha'-\eta'),\Re(\eta-\alpha)\}$. Then we have the following formula:

$$\{D_{0+}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\upsilon-1}S_n^m\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{\lambda}(at)\right)\}(x)=\frac{x^{\upsilon-\delta+\alpha+\alpha'-1}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^s$$

$$\times{}_4\psi_4\left[\begin{matrix}(\lambda,1),(\upsilon+\beta s,1),(\upsilon+\beta s-\delta+\alpha+\alpha'+\eta',1),(\upsilon+\beta s+\alpha-\eta,1)\\(\upsilon,\mu),(\upsilon+\beta s-\delta+\alpha+\alpha',1),(\upsilon+\beta s-\delta+\alpha+\eta',1),(\upsilon+\beta s-\eta,1)\end{matrix};ax\right]\quad(3.24)$$

Theorem 4 Let $\alpha,\alpha',\eta,\eta',\delta,\mu,\upsilon,\lambda,p\in C,\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>1,\ p\geq 0,t,x>0,$ and $a,\ \omega\in R$ such that $\Re(\delta)>0$ and $\Re(1-\delta-\upsilon)<1+min\{\Re(\eta'),\Re(\delta-\alpha-\alpha'),\Re(\delta-\alpha'-\eta)\}$. Then the right-sided generalized fractional differentiation $D_{-}^{\alpha,\alpha',\eta,\eta',\delta}$ of the product of the general class of polynomials $S_n^m[z]$ and extended Mittag-Leffler type function $\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(z;p)$ is given by

$$\{D_{-}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\delta-\upsilon}S_n^m\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}\left(at^{-1};p\right)\right)\}(x)=\frac{x^{-\upsilon+\alpha+\alpha'}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^s$$

$$\times{}_4\psi_4^{\left(\{k_\ell\}_{\ell\in N_0}\right)}\left[\begin{matrix}(\lambda,1),(\upsilon-\beta s-\alpha-\alpha',1),(\upsilon-\beta s-\alpha'-\eta,1),(\upsilon-\beta s-\delta+\eta',1)\\(\upsilon,\mu),(\upsilon-\beta s-\delta,1),(\upsilon-\beta s-\alpha-\alpha'-\eta,1),(\upsilon-\beta s-\delta-\alpha'+\eta',1)\end{matrix};\left(ax^{-1};p\right)\right]$$

Proof: Using (1.4) and (1.12), writing the function in series form, and applying (2.5) and (2.16), the left-hand side of (3.25) can be written as

$$\{D_{-}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\delta-\upsilon}S_n^m\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{\{k_\ell\}_{\ell\in N_0};\lambda}\left(at^{-1};p\right)\right)\}(x)$$

$$=\left\{D_{-}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\delta-\upsilon}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega t^{\beta}\right)^s\times\sum_{r=0}^{\infty}\frac{\mathrm{B}^{\left(\{k_\ell\}_{\ell\in N_0}\right)}(\lambda+r,1-\lambda;p)}{\mathrm{B}(\lambda,1-\lambda)}\times\frac{\left(at^{-1}\right)^r}{\Gamma(\mu r+\upsilon)}\right)\right\}(x),$$

By interchanging the order of differentiation and summations, which is valid under the conditions of Theorem 4, we have

$$=\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\ \omega^s\times\sum_{r=0}^{\infty}\frac{\mathrm{B}^{\left(\{k_\ell\}_{\ell\in N_0}\right)}(\lambda+r,1-\lambda;p)}{\mathrm{B}(\lambda,1-\lambda)}\times\frac{a^r}{\Gamma(\mu r+\upsilon)}\left(D_{-}^{\alpha,\alpha',\eta,\eta',\delta}t^{\delta-\upsilon+\beta s-r}\right)(x)$$

$$=\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\,\omega^{s}\times\sum_{r=0}^{\infty}\frac{\mathrm{B}^{\left(\{k_\ell\}_{\ell\in N_0}\right)}(\lambda+r,1-\lambda;p)}{\mathrm{B}(\lambda,1-\lambda)}\times\frac{a^{r}}{\Gamma(\mu r+\upsilon)}\left(I_{-}^{-\alpha',-\alpha,-\eta',-\eta,-\delta}t^{(1+\delta-\upsilon+\beta s-r)-1}\right)(x)$$

$$=\frac{x^{-\upsilon+\alpha+\alpha'}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^{s}\times\sum_{r=0}^{\infty}\frac{\mathrm{B}^{\left(\{k_\ell\}_{\ell\in N_0}\right)}(\lambda+r,1-\lambda;p)}{\mathrm{B}(\lambda,1-\lambda)}$$

$$\times\frac{\Gamma(\upsilon-\beta s-\alpha-\alpha'+r)\Gamma(\upsilon-\beta s-\alpha'-\eta+r)\Gamma(\upsilon-\beta s-\delta+\eta'+r)}{\Gamma(\upsilon-\beta s-\delta+r)\Gamma(\upsilon-\beta s-\alpha-\alpha'-\eta+r)\Gamma(\upsilon-\beta s-\delta-\alpha'+\eta'+r)}\times\frac{\left(ax^{-1}\right)^{r}}{\Gamma(\mu r+\upsilon)},\quad(3.26)$$

Now, by interpreting the terms of the above equation (3.26), in view of definition (1.9), we arrive at the result (3.25), which completes the proof of Theorem 4.

On setting $n=0, A_{0,0}=1$ then $S_{0}^{m}[z]\to 1$ in equation (3.25), we get the following result:

Corollary 4.1 Let $\alpha,\alpha',\eta,\eta',\delta,\mu,\upsilon,\lambda,p\in C,\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>1,p\geq 0,t,x>0,$ and $a,\omega\in R$ such that $\Re(\delta)>0$ and $\Re(1-\delta-\upsilon)<1+min\{\Re(\eta'),\Re(\delta-\alpha-\alpha'),\Re(\delta-\alpha'-\eta)\}$. Then we have the following formula:

$$\{D_{-}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\delta-\upsilon}\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}\left(at^{-1};p\right)\right)\}(x)$$

$$=\frac{x^{-\upsilon+\alpha+\alpha'}}{\Gamma(\lambda)}\times{}_4\psi_4^{\left(\{k_\ell\}_{\ell\in N_0}\right)}\left[\begin{matrix}(\lambda,1),(\upsilon-\alpha-\alpha',1),(\upsilon-\alpha'-\eta,1),(\upsilon-\delta+\eta',1)\\(\upsilon,\mu),(\upsilon-\delta,1),(\upsilon-\alpha-\alpha'-\eta,1),(\upsilon-\delta-\alpha'+\eta',1)\end{matrix};\left(ax^{-1};p\right)\right]\quad(3.27)$$

If we take $\alpha=\alpha+\eta,\alpha'=\eta'=0,\eta=-\delta$ and $\delta=\alpha$, Theorem 4 reduces to the following corollary:

Corollary 4.2 Let $\alpha,\eta,\delta,\mu,\upsilon,\lambda,p\in C,\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>1,p\geq 0,t,x>0$ and $a,\omega\in R$ such that $\Re(\alpha)>0$ and $\Re(1-\delta-\upsilon)<1+min\{\Re(-\eta),\Re(-\delta)\}$. Then we have

$$\{D_{-}^{\alpha,\eta,\delta}\left(t^{\alpha-\upsilon}S_{n}^{m}\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}\left(at^{-1};p\right)\right)\}(x)=\frac{x^{-\upsilon+\alpha+\eta}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^{s}$$

$$\times{}_3\psi_3^{\left(\{k_\ell\}_{\ell\in N_0}\right)}\left[\begin{matrix}(\lambda,1),(\upsilon-\beta s-\alpha-\eta,1),(\upsilon-\beta s+\delta,1)\\(\upsilon,\mu),(\upsilon-\beta s-\alpha,1),(\upsilon-\beta s+\delta-\alpha-\eta,1)\end{matrix};\left(ax^{-1};p\right)\right]\quad(3.28)$$

Now, if we take $\eta=-\alpha$ in Corollary 4.2, we arrive at the following result:

Corollary 4.3 Let $\alpha,\mu,\upsilon,\lambda,p\in C,\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>1,p\geq 0,t,x>0$ and $a,\omega\in R$ such that $\Re(\alpha)>0$. Then the following result holds:

$$\{D_{-}^{\alpha}\left(t^{\alpha-\upsilon}S_n^m\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}\left(at^{-1};p\right)\right)\}(x)$$

$$=\frac{x^{-\upsilon}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^{s}\times{}_2\psi_2^{\left(\{k_\ell\}_{\ell\in N_0}\right)}\left[\begin{matrix}(\lambda,1),(\upsilon-\beta s,1)\\(\upsilon,\mu),(\upsilon-\beta s-\alpha,1)\end{matrix};\left(ax^{-1};p\right)\right] \tag{3.29}$$

Next, if we put $k_\ell=\frac{(\rho)_\ell}{(\sigma)}(\ell\in N_0)$ in (3.25), then $\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(z;p)$ reduce to $\mathrm{E}_{\mu,\upsilon}^{(\rho,\sigma);\lambda}(z;p)$, we have the following formula:

Corollary 4.4 Let $\alpha,\alpha',\eta,\eta',\delta,\rho,\sigma\mu,\upsilon,\lambda,p\in C,\Re(\rho)>0,\Re(\sigma)>0,\Re(\mu)>0,\Re(\upsilon)>0,$ $\Re(\lambda)>1,p\geq 0,t,x>0,$ and $a,\omega\in R$ such that $\Re(\delta)>0$ and $\Re(1-\delta-\upsilon)<1+min\{\Re(\eta'),\Re(\delta-\alpha-\alpha'),\Re(\delta-\alpha'-\eta)\}$. Then the following result holds:

$$\{D_{-}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\delta-\upsilon}S_n^m\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{(\rho,\sigma);\lambda}\left(at^{-1};p\right)\right)\}(x)=\frac{x^{-\upsilon+\alpha+\alpha'}\Gamma(\sigma)}{\Gamma(\lambda)\Gamma(\rho)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^{s}$$

$$\times{}_5\psi_5\left[\begin{matrix}(\lambda,1),(\upsilon-\beta s-\alpha-\alpha',1),(\upsilon-\beta s-\alpha'-\eta,1),(\upsilon-\beta s-\delta+\eta',1),(\rho,1)\\(\upsilon,\mu),(\upsilon-\beta s-\delta,1),(\upsilon-\beta s-\alpha-\alpha'-\eta,1),(\upsilon-\beta s-\delta-\alpha'+\eta',1),(\sigma,1)\end{matrix};\left(ax^{-1};p\right)\right]. \tag{3.30}$$

Again, if we set $k_\ell=1(\ell\in N_0)$ in (3.25), then $\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(z;p)$ reduce to $\mathrm{E}_{\mu,\upsilon}^{\lambda}(z;p)$, we obtain the following result:

Corollary 4.5 Let $\alpha,\alpha',\eta,\eta',\delta,\mu,\upsilon,\lambda,p\in C,\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>1,p\geq 0,t,x>0,$ and $a,\omega\in R$ such that $\Re(\delta)>0$ and $\Re(1-\delta-\upsilon)<1+min\{\Re(\eta'),\Re(\delta-\alpha-\alpha'),\Re(\delta-\alpha'-\eta)\}$. Then the following formula holds:

$$\{D_{-}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\delta-\upsilon}S_n^m\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{\lambda}\left(at^{-1};p\right)\right)\}(x)=\frac{x^{-\upsilon+\alpha+\alpha'}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^{s}$$

$$\times{}_4\psi_4\left[\begin{matrix}(\lambda,1),(\upsilon-\beta s-\alpha-\alpha',1),(\upsilon-\beta s-\alpha'-\eta,1),(\upsilon-\beta s-\delta+\eta',1)\\(\upsilon,\mu),(\upsilon-\beta s-\delta,1),(\upsilon-\beta s-\alpha-\alpha'-\eta,1),(\upsilon-\beta s-\delta-\alpha'+\eta',1)\end{matrix};\left(ax^{-1};p\right)\right]. \tag{3.31}$$

Further, if we take $k_\ell=1(\ell\in N_0)$ *and* $p=0$ in (3.25), then $\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(z;p)$ reduce to $\mathrm{E}_{\mu,\upsilon}^{\lambda}(z)$, then we obtain the following result:

Corollary 4.6 Let $a,\alpha,\alpha',\eta,\eta',\delta,\mu,\upsilon,\lambda \in C, \Re(\mu)>0, \Re(\upsilon)>0, \Re(\lambda)>1, t,x>0,$ and $a,\omega \in R$ such that $\Re(\delta)>0$ and $\Re(1-\delta-\upsilon)<1+min\{\Re(\eta'), \Re(\delta-\alpha-\alpha'), \Re(\delta-\alpha'-\eta)\}$. Then we have holds the following formula:

$$\{D_{-}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\delta-\upsilon}S_n^m\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{\lambda}\left(at^{-1}\right)\right)\}(x)=\frac{x^{-\upsilon+\alpha+\alpha'}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^{s}$$

$$\times{}_4\psi_4\left[\begin{matrix}(\lambda,1),(\upsilon-\beta s-\alpha-\alpha',1),(\upsilon-\beta s-\alpha'-\eta,1),(\upsilon-\beta s-\delta+\eta',1)\\(\upsilon,\mu),(\upsilon-\beta s-\delta,1),(\upsilon-\beta s-\alpha-\alpha'-\eta,1),(\upsilon-\beta s-\delta-\alpha'+\eta',1)\end{matrix};ax^{-1}\right]. \tag{3.32}$$

4. Conclusion

This study establishes four principal image formulae for generalized fractional calculus operators acting on the product of Srivastava polynomials and an extended Mittag-Leffler type function. Two formulae concern left-sided and right-sided generalized fractional integration, while the remaining two address the corresponding generalized fractional differentiation operators. In each case, the resulting expressions are represented in terms of the generalized Wright hypergeometric function under the stated parameter restrictions. The corollaries obtained through appropriate choices of parameters and the defining coefficient sequence show how the principal results reduce to forms involving related Mittag-Leffler type functions and to cases associated with Saigo and Riemann-Liouville fractional calculus operators. The general class of Srivastava polynomials also permits further specialisation to familiar polynomial families described in the manuscript. These results therefore provide a unified theoretical representation for the fractional integration and differentiation of the specified product. Further work may examine applications of the established formulae to fractional differential equations and related mathematical-physics problems, subject to the corresponding analytical conditions.

5. Limitations

The study is theoretical and is restricted to the generalized fractional calculus operators, parameter conditions, convergence requirements, Srivastava polynomials, and extended Mittag-Leffler type function considered in the manuscript. The results are presented as analytical image formulae and corollaries; no numerical examples, computational validation, or explicit solutions of fractional differential equations are developed. Consequently, applications to particular physical or engineering models require separate verification of the relevant parameter restrictions and convergence conditions.

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