On Contra-continuous Functions in Ideal Topological Space

Original Research Article

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Abstract

Aims/objectives: The aim of this paper is to introduce a class of functions using the concepts in ideal topological space called the contra $R-\mathcal{I}-$continuous functions. The paper also introduced almost contra $R-\mathcal{I}-$continuous functions and investigated certain properties and several characterizations of such concepts. Further, contra $R-\mathcal{I}-$closed graphs and strongly contra $R-\mathcal{I}-$closed graphs are dealt with.
Study design: Theoretical
Place and Duration of Study: Department of Mathematics, St. Joseph's College (Autonomous) Devagiri, Calicut-673008, India.
Methodology:The application of logical reasoning to deduce new theorems from established principles.
Conclusion: This paper introduced and systematically investigated the concepts of contra $R-\mathcal{I}-$continuous and almost contra $R-\mathcal{I}-$continuous functions in ideal topological spaces. It provides analysis of their fundamental properties, characterizations, and relationships with other existing continuous functions. Furthermore, it examined the behaviors of contra $R-\mathcal{I}-$closed graphs and strongly contra $R-\mathcal{I}-$closed graphs.

Keywords: contra $R-\mathcal{I}-$continuous functions, almost contra $R-\mathcal{I}-$continuous functions, contra $R-\mathcal{I}-$closed graphs, strongly contra $R-\mathcal{I}-$closed graphs.

2010 Mathematics Subject Classification: 54A05

1 Introduction

The study of continuity and its generalizations, separation axioms, is a very active and fruitful area of research in general topology. One of the strong and prominent kinds of classical continuity is contra-continuity, which was first introduced by Dontchev in 1996 (Dontchev, J. 1996). Following this seminal work, various researchers like S. Jafari and T. Noiri (Jafari, S., & Noiri, T. 1999), along with E. Ekici (Ekici, E. 2004), have investigated and formulated different weak forms of contra-continuity. Besides these developments, the embedding of ideals in topological spaces has given a powerful setting for extending classical topological operators and structural properties. The concept of ideal and the corresponding local function were first introduced by Kuratowski and Vaidyanathaswamy (Kuratowski, 1966; Vaidyanathaswamy, 1960) and later systematically developed by Jankovic and Hamlett (Jankovic, D., & Hamlett, T. R. 1990). Recently, ideals have been used to describe generalized open sets and to study algebraic associations, primals, filters, and idealized expansions of open sets (Matejdes, 2024). In particular, Sangeetha M.V. & Baby Chacko (Sangeetha M.V. & Baby Chacko, 2019 and 2020) defined and studied the ideal topological spaces via the concept of $R-\mathcal{I}-$open sets and discussed supra spaces and weak separation axioms. The main goal of this paper is to combine these lively directions of research. Using the notion of $R-\mathcal{I}-$open sets, we define and study some new classes of generalized contra-continuous maps. In section 3 of this paper, using the notion of $R-\mathcal{I}-$open sets, the contra $R-\mathcal{I}-$continuous functions are introduced and studied. In section 4, almost contra $R-\mathcal{I}-$continuous functions are investigated. Section 5 deals with $R-\mathcal{I}-$closed, contra $R-\mathcal{I}-$closed and strongly contra $R-\mathcal{I}-$closed graphs.

2 Preliminaries

By a space (X,τ), we mean a topological space with a topology τ defined on X on which no separation axioms are assumed unless otherwise explicitly stated. For a given point x in a space (X,τ), the system of open neighborhoods of x is denoted by $N(x)=\{U\in\tau : x\in U\}$. For a given subset A of a space (X,τ), $Cl(A)$ and $Int(A)$ are used to denote the closure of A and the interior of A, respectively, with respect to the topology.

A nonempty collection of subsets of a set X is said to be an ideal $\mathcal{I}$ on X, if it satisfies the following two conditions: (i) If $A\in\mathcal{I}$ and $B\subset A$, then $B\in\mathcal{I}$; (ii) If $A\in\mathcal{I}$ and $B\in\mathcal{I}$, then $A\cup B\in\mathcal{I}$. An ideal topological space (or ideal space) $(X,\tau,\mathcal{I})$ means a topological space (X,τ) with an ideal $\mathcal{I}$ defined on X. Let (X,τ) be a topological space with an ideal $\mathcal{I}$ defined on X. Then for any subset A of X, $A^*(\mathcal{I},\tau)=\{x\in X/A\cap U\notin\mathcal{I}\ for\ every\ \ U\in N(x)\}$ is called the local function of A with respect to $\mathcal{I}$ and τ. If there is no ambiguity, we will write $A^*(\mathcal{I})$ or simply A^* for $A^*(\mathcal{I},\tau)$. Also, $Cl^*(A)=A\cup A^*$. It defines a Kuratowski closure operator for the topology $\tau^*(\mathcal{I})$ (or simply τ^*) which is finer than τ (Jankovic, D., & Hamlett, T. R. 1990).

A subset A of an ideal topological space $(X,\tau,\mathcal{I})$ is said to be $R-\mathcal{I}-$**open** (resp. regular open) if $Int(Cl^*(A))=A$ (resp. $Int(Cl(A))=A$). We call a subset A of $(X,\tau,\mathcal{I})$ is $R-\mathcal{I}-$closed if its complement is $R-\mathcal{I}-$open. The intersection of all $R-\mathcal{I}-$closed sets containing A is called the $R-\mathcal{I}-$closure of A and is denoted by $R-\mathcal{I}-$Cl(A). The $R-\mathcal{I}-$interior of A is defined by the union of all $R-\mathcal{I}-$open sets contained in A and is denoted by $R-\mathcal{I}-$Int(A). The family of all $R-\mathcal{I}-$open (resp. $R-\mathcal{I}-$closed) sets of $(X,\tau,\mathcal{I})$ containing a point $x\in X$ is denoted by $R\mathcal{I}O(X,x)$ (resp. $R\mathcal{I}C(X,x)$). A subset N of X is called an $R-\mathcal{I}-$nbd of a point $x\in X$ if there exists an $R-\mathcal{I}-$open set U of $(X,\tau,\mathcal{I})$ so that $x\in U\subseteq N$ (Sangeetha, M. V., & Chacko, B. 2020a).

Definition 2.1. (Prasannan, A. R., & Biswas, J. 2016) A function $f:(X,\tau,\mathcal{I})\to(Y,\sigma)$ is said to be $R-\mathcal{I}-$continuous if for each $x\in X$ and each open set $V\in\sigma$ containing $f(x)$, there exists an $R-\mathcal{I}-$open set $U\subset X$ such that $x\in U$ and $f(U)\subset V$.

Definition 2.2. (Mrsevic, M. 1986) Let A be a subset of an ideal topological space $(X,\tau,\mathcal{I})$. Then the set $\cap\{U\in\tau : A\subset U\}$. is called the kernel of A and denoted by $Ker(A)$.

Lemma 2.1. *(Jafari, S., & Noiri, T. 1999) Let A be a subset of an ideal topological space $(X,\tau,\mathcal{I})$.*

1. *$x\in Ker(A)$ if and only if $A\cap F\neq\phi$ for any closed subset F of X containing x*
2. *$A\subset Ker(A)$ and $A=Ker(A)$ if A is open in X*
3. *if $A\subset B$, then $Ker(A)\subset Ker(B)$*

Lemma 2.2. *The following properties hold for a subset A of an ideal topological space $(X, \tau, \mathcal{I})$.*

1. *$R - \mathcal{I} - Int(A) = X \setminus R - \mathcal{I} - Cl(X - A)$.*
2. *$x \in R - \mathcal{I} - Cl(A)$ if and only if $A \cap U \neq \phi$ for each $U \in R\mathcal{I}O(X, x)$*

3 On contra $R - \mathcal{I}-$continuous Functions

Definition 3.1. A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is called a contra $R - \mathcal{I}-$continuous function if $f^{-1}(V)$ is $R - \mathcal{I}-$closed in X for every open set V in Y.

Example 3.1. *Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$, $\sigma = \{\phi, X, \{a\}, \{a, b\}\}$, $\mathcal{I} = \{\phi, \{a\}\}$. Let $f : (X, \tau, \mathcal{I}) \to (X, \sigma)$ be such that $f(a) = c, f(b) = a, f(c) = a$. Then f is contra $R - \mathcal{I}-$continuous.*

Example 3.2. *Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$, $\sigma = \{\phi, X, \{a\}, \{a, b\}\}$, $\mathcal{I} = \{\phi, \{a\}\}$. Let $g : (X, \tau, \mathcal{I}) \to (X, \sigma)$ be such that $g(a) = a, g(b) = b, g(c) = c$. Then g is contra continuous but not contra $R - \mathcal{I}-$continuous.*

Theorem 3.3. *For a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ the following conditions are equivalent.*

1. *f is contra $R - \mathcal{I}-$continuous*
2. *for each $x \in X$ and each closed set F in Y containing $f(x)$ there exists a $R - \mathcal{I}-$open set U containing x such that $f(U) \subset F$*
3. *for each $x \in X$ and each closed set F in Y containing $f(x)$, $f^{-1}(F)$ is $R - \mathcal{I}-$open in X*
4. *$f(R - \mathcal{I} - Cl(A)) \subset Ker(f(A))$ for every $A \subset X$*
5. *$(R - \mathcal{I} - Cl(f^{-1}(B))) \subset f^{-1}(Ker(B))$ for every $B \subset Y$*

Proof.
1. $\Rightarrow$ 2.
Let $x \in X$ and F be any closed subset of Y containing $f(x)$. Then by the contra $R - \mathcal{I}-$continuity of f, $f^{-1}(Y \setminus F) = X \setminus f^{-1}(F)$ is $R - \mathcal{I}-$closed in X. Hence, by taking $U = f^{-1}(F)$, we get there exists a $R - \mathcal{I}-$open set U containing x such that $f(U) \subset F$.
2. $\Rightarrow$ 3.
Let F be any closed subset of Y and let $x \in f^{-1}(F)$. Then $f(x) \in F$ and for each $x \in f^{-1}(F)$ there exists a $R - \mathcal{I}-$open set U_x containing x such that $f(U_x) \subset F$. Hence $f^{-1}(F) = \cup\{U_x : x \in f^{-1}(F)\}$. So, $f^{-1}(F)$ is $R - \mathcal{I}-$open in X.

We use Lemma 1 in all the following implications.
3. $\Rightarrow$ 4.
Let $A \subset X$ and assume that $y \notin Ker(f(A))$. Then there exists a closed set F in Y with $y \in F$ such that $f(A) \cap F = \phi$. This implies $A \cap f^{-1}(F) = \phi$ and so $R - \mathcal{I} - Cl(A) \subset X \setminus f^{-1}(F)$. Thus, $R - \mathcal{I} - Cl(A) \cap f^{-1}(F) = \phi$ and we obtain $f(R - \mathcal{I} - Cl(A)) \cap F = \phi$. So $y \notin f(R - \mathcal{I} - Cl(A))$. Hence $f(R - \mathcal{I} - Cl(A)) \subset Ker(f(A))$ for every $A \subset X$.
4. $\Rightarrow$ 5.
Let $B \subset Y$. Then $f(R - \mathcal{I} - Cl(f^{-1}(B))) \subset Ker(f(f^{-1}(B))) \subset Ker(B)$. Thus, $(R - \mathcal{I} - Cl(f^{-1}(B))) \subset f^{-1}(Ker(B))$ for every $B \subset Y$.
5. $\Rightarrow$ 1.
Let V be an open set in Y. Then $(R - \mathcal{I} - Cl(f^{-1}(V))) \subset f^{-1}(Ker(V)) = f^{-1}(V)$ and so $(R - \mathcal{I} - Cl(f^{-1}(V))) = f^{-1}(V)$. Thus, $f^{-1}(V)$ is $R - \mathcal{I}-$closed in X and hence, f is contra $R - \mathcal{I}-$continuous.

Theorem 3.4. *Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be a function which is contra $R - \mathcal{I}-$continuous where Y is a regular space; then f is $R - \mathcal{I}-$continuous.*

Proof.
Let $x \in X$ and V be an open set in Y with $f(x) \in V$. By the regularity of Y, there exists an open set H in Y containing $f(x)$ such that $Cl(H) \subset V$. Also, there exists a $R-\mathcal{I}-$open set $U \in X$ containing x such that $f(U) \subset Cl(H)$, since f is contra $R-\mathcal{I}-$continuous. Thus, $f(U) \subset V$ and f is $R-\mathcal{I}-$continuous.

Definition 3.2. (Sangeetha, M. V., & Chacko, B. 2020a) A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ is called $R-\mathcal{I}-$open if the image of each $R-\mathcal{I}-$open set in X is $R-\mathcal{J}-$open in Y.

Definition 3.3. (Sangeetha, M. V., & Chacko, B. 2020a) A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ is $R^*-\mathcal{I}-$irresolute if the inverse image of every $R-\mathcal{J}-$open set in Y is $R-\mathcal{I}-$open in X.

Definition 3.4. An ideal topological space $(X, \tau, \mathcal{I})$ is said to be $R-\mathcal{I}-$connected if X cannot be written as the union of two non-empty disjoint $R-\mathcal{I}-$open subsets of X.

Definition 3.5. The $R-\mathcal{I}-$frontier of a subset A of a space X, denoted by $R-\mathcal{I}-\partial$(A) and is defined as $R-\mathcal{I}-\partial(A) = R-\mathcal{I}-Cl(A) \cap R-\mathcal{I}-Cl(X \setminus A)$.

Definition 3.6. A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ is called almost-$R-\mathcal{I}-$continuous if, for each $x \in X$ and for each open set $V \in Y$ containing $f(x)$, there exists a $R-\mathcal{I}-$open set $U \in X$ containing x such that $f(U) \subset R-\mathcal{J}-Int(Cl(V))$.

Definition 3.7. A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is called almost weakly-$R-\mathcal{I}-$continuous if, for each $x \in X$ and for each open set $V \in Y$ containing $f(x)$, there exists a $R-\mathcal{I}-$open set $U \in X$ containing x such that $f(U) \subset Cl(V)$.

Theorem 3.5. *Let X be a $R-\mathcal{I}-$connected space. If $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is contra $R-\mathcal{I}-$continuous and surjective, then Y is connected.*

Proof.
Assume that Y is disconnected. Then $Y = A \cup B$, where A and B are disjoint non-empty clopen subsets of Y. Since, f is contra $R-\mathcal{I}-$continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are non-empty $R-\mathcal{I}-$clopen sets in X. Also $f^{-1}(A) \cup f^{-1}(B) = f^{-1}(A \cup B) = X$ and $f^{-1}(A) \cap f^{-1}(B) = f^{-1}(A \cap B) = \phi$. This contradicts X is $R-\mathcal{I}-$connected, and so Y is connected.

Theorem 3.6. *If $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ is $R-\mathcal{I}-$open and contra-$R-\mathcal{I}-$continuous, then f is almost-$R-\mathcal{I}-$continuous.*

Proof.
Let $x \in X$ and $V \in Y$ be an open set containing $f(x)$. Since f is contra $R-\mathcal{I}-$continuous, there exists a $R-\mathcal{I}-$open set $U \in X$ containing x such that $f(U) \subset Cl(V)$. Since f is $R-\mathcal{I}-$open, $f(U)$ is $R-\mathcal{J}-$open in Y. So $f(U) = R-\mathcal{J}-Int(f(U))$. Hence $f(U) \subset R-\mathcal{J}-Int(Cl(V))$ and f is almost-$R-\mathcal{I}-$continuous.

Theorem 3.7. *If a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is contra $R-\mathcal{I}-$continuous then f is almost weakly-$R-\mathcal{I}-$continuous.*

Proof.
Let $x \in X$ and $V \in Y$ be an open set containing $f(x)$. Since f is contra $R-\mathcal{I}-$continuous, $f^{-1}(Cl(V))$ is $R-\mathcal{I}-$open in X. Take $U = f^{-1}(Cl(V))$. Then $f(U) \subset Cl(V)$. Thus f is almost weakly-$R-\mathcal{I}-$continuous.

Theorem 3.8. *The set of all points $x \in X$ at which $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is not contra-$R-\mathcal{I}-$continuous is identical with the union of $R-\mathcal{I}-$frontier of the inverse images of closed sets of Y containing $f(x)$.*

Proof.
First assume that f is not contra $R-\mathcal{I}-$continuous at $x \in X$. Then there exists a closed set $F \subset Y$ containing $f(x)$ such that $f(U) \cap (Y \setminus F) \neq \phi$ for every $R-\mathcal{I}-$open set $U \subset X$ containing x. So $U \cap f^{-1}(Y \setminus F) \neq \phi$. Thus $x \in R-\mathcal{I}-Cl(f^{-1}(Y \setminus F)) = R-\mathcal{I}-Cl(X \setminus f^{-1}(F))$. Since $x \in f^{-1}(F)$, $x \in R-\mathcal{I}-Cl(f^{-1}(F))$. This implies $x \in R-\mathcal{I}-\partial(f^{-1}(F))$.
Now suppose that $x \in R-\mathcal{I}-\partial(f^{-1}(F))$ for some closed set $F \subset Y$ containing $f(x)$. Assume that f is contra $R-\mathcal{I}-$continuous at x. Then there exists a $R-\mathcal{I}-$open set $U \subset X$ containing x such that $f(U) \subset F$. This implies $x \in U \subset f^{-1}(F)$. So $x \notin R-\mathcal{I}-\partial(f^{-1}(F))$, a contradiction. Thus, f is not contra $R-\mathcal{I}-$continuous at x.

Theorem 3.9. *Let $f:(X,\tau,\mathcal{I}) \to (Y,\sigma,\mathcal{J})$ be a contra $R-\mathcal{I}-$continuous function and $g:(Y,\sigma,\mathcal{J}) \to (Z,\eta)$ be a continuous function. Then $g \circ f:(X,\tau,\mathcal{I}) \to (Z,\eta)$ is contra $R-\mathcal{I}-$continuous.*

Proof.
Let $x \in X$ and $W \in Z$ be a closed set containing $(g \circ f)(x)$. Then $V = g^{-1}(W)$ is closed in Y by the continuity of g. Also, since f is contra $R-\mathcal{I}-$continuous, there exists a $R-\mathcal{I}-$open set $U \in X$ containing x such that $f(U) \subset V = g^{-1}(W)$. Thus $(g \circ f)(U) \subset W$. Hence $g \circ f:(X,\tau,\mathcal{I}) \to (Z,\eta)$ is contra $R-\mathcal{I}-$continuous.

Theorem 3.10. *Let $f:(X,\tau,\mathcal{I}) \to (Y,\sigma,\mathcal{J})$ be a $R^*-\mathcal{I}-$irresolute function and $g:(Y,\sigma,\mathcal{J}) \to (Z,\eta)$ be a contra $R-\mathcal{I}-$continuous function. Then $g \circ f:(X,\tau,\mathcal{I}) \to (Z,\eta)$ is contra $R-\mathcal{I}-$continuous.*

Proof.
Let $x \in X$ and $W \in Z$ be a closed set containing $(g \circ f)(x)$. Since g is contra $R-\mathcal{I}-$continuous, there exists a $R-\mathcal{J}-$open set $V \in Y$ containing $f(x)$ such that $g(V) \subset W$. Also, since f is $R^*-\mathcal{I}-$irresolute, $f^{-1}(V)$ is $R-\mathcal{I}-$open in X containing x. Take $U = f^{-1}(V)$. Thus $(g \circ f)(U) \subset W$. Hence $g \circ f:(X,\tau,\mathcal{I}) \to (Z,\eta)$ is contra $R-\mathcal{I}-$continuous.

Theorem 3.11. *Let $f:(X,\tau,\mathcal{I}) \to (Y,\sigma,\mathcal{J})$ be a surjective $R^*-\mathcal{I}-$irresolute and $R-\mathcal{I}-$open function and $g:(Y,\sigma,\mathcal{J}) \to (Z,\eta)$ be any function. Then $g \circ f:(X,\tau,\mathcal{I}) \to (Z,\eta)$ is contra $R-\mathcal{I}-$continuous if and only if g is contra $R-\mathcal{I}-$continuous.*

Proof.
If g is contra $R-\mathcal{I}-$continuous, then the proof is clear. Now assume $g \circ f$ is contra $R-\mathcal{I}-$continuous. Let W be closed in Z. Then $(g \circ f)^{-1}(W)$ is $R-\mathcal{I}-$open in X. Since f is $R-\mathcal{I}-$open, $f(f^{-1}(g^{-1}(W)))$ is $R-\mathcal{J}-$open in Y. Hence $g^{-1}(W)$ is $R-\mathcal{J}-$open in Y and so g is contra $R-\mathcal{I}-$continuous.

Definition 3.8. (Sangeetha, M. V., & Chacko, B. 2020b) A topological space $(X,\tau,\mathcal{I})$ is said to be $R-\mathcal{I}-T_1$ if for each pair of distinct points x and y in X, there exists $R-\mathcal{I}-$open sets U and V of X, such that $x \in U, y \notin U$ and $y \in V, x \notin V$.

Definition 3.9. (Sangeetha, M. V., & Chacko, B. 2020b) A topological space $(X,\tau,\mathcal{I})$ is said to be $R-\mathcal{I}-T_2$ if for each pair of distinct points x and y in X, there exists disjoint $R-\mathcal{I}-$open sets U and V in X such that $x \in U$and $y \in V$.

Definition 3.10. (Sangeetha, M. V., & Chacko, B. 2020b) An ideal topological space X is $R-\mathcal{I}-$normal if for each pair A, B of disjoint $R-\mathcal{I}-$closed sets of X, there exists disjoint $R-\mathcal{I}-$open sets containing A and B respectively.

Definition 3.11. (Staum, R. 1974) A space (X,τ) is said to be ultra normal if each pair of non-empty disjoint closed sets can be separated by disjoint clopen sets.

Definition 3.12. A topological space (X, τ) is ultra Hausdorff (Staum, R. 1974) if for each pair of distinct points x and y of X there exist closed sets U and V such that $x \in U, y \in V\ and\ U \cap V = \phi$. A topological space (X, τ) is said to be weakly Hausdorff (Soundararajan, T. 1971) if each element of X is the intersection of regular closed sets of X.

Definition 3.13. (Singal, M. K., & Mathur, A. 1969) A topological space (X, τ) is called Urysohn if, for each $x, y \in X$ with $x \neq y$, there exist open sets P and Q of X containing x and y respectively such that $Cl(P) \cap Cl(Q) = \phi$.

Theorem 3.12. *Let (X, τ, I) be a $R-\mathcal{I}-$connected space and (Y, σ) a T_1 space. If $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is contra $R-\mathcal{I}-$continuous, then f is constant.*

Proof.
Let X be a $R-\mathcal{I}-$connected space and Y be a T_1 space. Then $\mathcal{P} = \{f^{-1}(y : y \in Y)\}$ is a disjoint collection of $R-\mathcal{I}-$open sets which partitions X. If $|\mathcal{P}| \geq 2$, then X is the union of two disjoint non-empty $R-\mathcal{I}-$open sets. This implies $|\mathcal{P}| = 1$ and hence f is constant.

Theorem 3.13. *Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be a contra $R-\mathcal{I}-$continuous and one-one function. If Y is a Urysohn space, then X is $R-\mathcal{I}-T_2$.*

Proof.
Let $x \neq y \in X$. Then $f(x) \neq f(y)$. Since Y is a Urysohn space, there exist open sets $P,\ Q \in Y$ such that $f(x) \in P,\ f(y) \in Q$ and $Cl(P) \cap Cl(Q) = \phi$. Since f is contra $R-\mathcal{I}-$continuous at there exist $R-\mathcal{I}-$open sets $U,\ V \in X$ such that $x \in U,\ y \in V$ and $f(U) \subset Cl(P),\ f(V) \subset Cl(Q)$. Thus $U \cap V = \phi$, since $f(U) \cap f(V) = \phi$. Hence X is $R-\mathcal{I}-T_2$.

Theorem 3.14. *Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ be a contra $R-\mathcal{I}-$continuous, $R-\mathcal{I}-$closed and one-one function where Y is ultra normal, then X is $R-\mathcal{I}-$normal.*

Proof.
Let $A,\ B \in X$ be disjoint closed subsets. Since f is $R-\mathcal{I}-$closed and one-one, $f(A)$ and $f(B)$ are disjoint $R-\mathcal{I}-$closed subsets in Y and hence closed subsets of Y. Since Y is ultra normal, $f(A)$ and $f(B)$ are separated by disjoint clopen sets, say, U and V. Since f is contra $R-\mathcal{I}-$continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are $R-\mathcal{I}-$open by theorem 3.3 (3). Also, $A \subset f^{-1}(U),\ B \subset f^{-1}(V)$. This gives $f^{-1}(U) \cap f^{-1}(V) = \phi$ and so X is $R-\mathcal{I}-$normal.

Theorem 3.15. *Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ be a contra $R-\mathcal{I}-$continuous and one-one function where Y is ultra Hausdorff, then X is $R-\mathcal{I}-T_2$.*

Proof.
Let $x \neq y \in X$. Then $f(x) \neq f(y)$. Since Y is ultra Hausdorff, there exists closed sets U and V in Y such that $f(x) \in U,\ f(y) \in V\ and\ U \cap V = \phi$. Since f is contra $R-\mathcal{I}-$continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are $R-\mathcal{I}-$open in X containing x and y respectively with $f^{-1}(U) \cap f^{-1}(V) = \phi$. Hence X is $R-\mathcal{I}-T_2$.

4 Almost contra $R-\mathcal{I}-$continuous Functions

Definition 4.1. A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is called almost contra $R-\mathcal{I}-$continuous if $f^{-1}(V)$ is $R-\mathcal{I}-$closed in $(X, \tau, \mathcal{I})$ for every regular open set V in (Y, σ).

Definition 4.2. An ideal topological space is said to be countably $R-\mathcal{I}-$compact if every countable $R-\mathcal{I}-$open cover admits a finite subcover.

Definition 4.3. An ideal topological space is said to be $R-\mathcal{I}-$closed compact ($R-\mathcal{I}-$closed Lindeloff) if every $R-\mathcal{I}-$closed cover admits a finite (countable) subcover.
An ideal topological space is said to be countably $R-\mathcal{I}-$closed compact if every countable $R-\mathcal{I}-$closed cover admits a finite subcover.

Definition 4.4. (Sangeetha, M. V., & Chacko, B. 2020b) An ideal topological space is said to be $R-\mathcal{I}-$compact ($R-\mathcal{I}-$Lindeloff) if every $R-\mathcal{I}-$open cover admits a finite (countable) subcover.

Definition 4.5. A topological space X is called S-closed (Joseph, J. E., & Kwack, M. H. 1980) (resp. countably S-closed (Dlaska, K., Ergun, N., & Ganster, M. 1994), S-Lindeloff (Ekici, E. 2004)) if every regular closed (resp. countably regular closed, regular closed) cover of X has a finite (resp. finite, countable) subcover.
A topological space X is said to be nearly compact (resp. nearly countably compact, nearly Lindeloff) if every regular open (resp. countable regular open, regular open) cover of X has a finite (resp. finite, a countably) subcover. (Singal, M. K., & Mathur, A. 1969)

Theorem 4.1. *The following statements are equivalent for a function $f:(X,\tau,\mathcal{I})\to(Y,\sigma)$:*

1. *f is almost contra $R-\mathcal{I}-$continuous*
2. *$f^{-1}(F)$ is $R-\mathcal{I}-$open in X for each regular closed set F in Y*
3. *for each $x\in X$ and each regular closed set F in Y containing $f(x)$ there exist a $R-\mathcal{I}-$open set U in X containing x such that $f(U)\subset F$*
4. *for each $x\in X$ and each regular open set V in Y not containing $f(x)$ there exist a $R-\mathcal{I}-$closed set C in X not containing x such that $f^{-1}(V)\subset C$*

Proof.
1. $\Leftrightarrow$ 2.
Let F be a regular closed set in Y. Then $f^{-1}(Y\setminus F)=X\setminus f^{-1}(F)$ is $R-\mathcal{I}-$closed in X. Hence $f^{-1}(F)$ is $R-\mathcal{I}-$open in X. In a similar manner, the converse part can be proved.
2. $\Leftrightarrow$ 3.
Let F be a regular closed set in Y containing $f(x)$. Then $f^{-1}(F)$ is $R-\mathcal{I}-$open in X and contains x. Take $U=f^{-1}(F)$. Hence U is a $R-\mathcal{I}-$open set in X containing x such that $f(U)\subset F$.
Let F be a regular closed set in Y and let $x\in f^{-1}(F)$. Then there exists a $R-\mathcal{I}-$open set U_x containing x such that $f(U_x)\subset F$. Hence $f^{-1}(F)=\cup\{U_x : x\in f^{-1}(F)\}$. So $f^{-1}(F)$ is $R-\mathcal{I}-$open in X.
3. $\Leftrightarrow$ 4.
Let $x\in X$ and V be a regular open set not containing $f(x)$. Then $Y\setminus V$ is a regular closed set containing $f(x)$. So there will exist a $R-\mathcal{I}-$open set $U\in X$ containing x such that $f(U)\subset(Y\setminus V)$. Hence $U\subset f^{-1}(Y\setminus V)\subset X\setminus f^{-1}(V)$. So $f^{-1}(V)\subset X\setminus U$. Taking $C=X\setminus U$ gives a $R-\mathcal{I}-$closed set C in X not containing x such that $f^{-1}(V)\subset C$. Similarly, the converse can be proved.

Theorem 4.2. *If $f:(X,\tau,\mathcal{I})\to(Y,\sigma)$ is almost contra $R-\mathcal{I}-$continuous, then f is almost weakly-$R-\mathcal{I}-$continuous.*

Proof.
Let $x\in X$ and V be an open set in Y containing $f(x)$. Obviously $Cl(V)$ is regular closed in Y containing $f(x)$. Then, since f is almost contra $R-\mathcal{I}-$continuous, there exist a $R-\mathcal{I}-$open set U in X containing x such that $f(U)\subset Cl(V)$. Hence f is almost weakly-$R-\mathcal{I}-$continuous.

Lemma 4.3. *A function $f:(X,\tau,\mathcal{I})\to(Y,\sigma,\mathcal{J})$ is almost $R-\mathcal{I}-$continuous, if and only if for each $x\in X$ and each regular open set V of Y containing $f(x)$, there exist a $R-\mathcal{I}-$open set $U\in X$ containing x such that $f(U)\subset V$.*

Proof.
The 'if part' is trivial. The 'only if' part can be proved by taking V as a open set in Y and hence $R - \mathcal{J} - Int(Cl(V))$ will be regular open .

Theorem 4.4. *Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ be a function where Y is extremally disconnected. Then f is almost contra $R - \mathcal{I}-$continuous if and only if f is almost $R - \mathcal{I}-$continuous.*
Note. A topological (X, τ) is extremally disconnected if the closure of every open set in X is open in X.

Proof.
Let $x \in X$ and $V \in Y$ be a regular open set containing $f(x)$. Now Y is extremally disconnected $\longrightarrow$ $Cl(V)$ is open $\longrightarrow Int(Cl(V)) = V \longrightarrow Cl(V) = V \longrightarrow V$ is regular closed. Since f is almost contra $R - \mathcal{I}-$continuous, there exist a $R - \mathcal{I}-$open set U in X containing x such that $f(U) \subset V$. Then by the above lemma, f is almost $R - \mathcal{I}-$continuous.
Conversely assume f is almost $R - \mathcal{I}-$continuous. Let $x \in X$ and $V \in Y$ be a regular closed set containing $f(x)$. This implies $Cl(Int(V)) = V$. Since Y is extremally disconnected, we get V is open in Y and thus is regular open in Y. Since f is almost $R - \mathcal{I}-$continuous, there exist a $R - \mathcal{I}-$open set $U \in X$ containing x such that $f(U) \subset V$ by lemma 3. Thus f is almost contra $R - \mathcal{I}-$continuous.

Theorem 4.5. *If $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is an almost contra $R - \mathcal{I}-$continuous and one-one function where Y is weakly Hausdorff, then X is $R - \mathcal{I} - T_1$.*

Proof.
Let $x \neq y \in X$. Since f is one-one, $f(x) \neq f(y)$ in Y. Since Y is weakly Hausdorff there exist disjoint regular closed sets C and D such that $f(x) \in C,\ f(y) \notin C$ and $f(y) \in D,\ f(x) \notin D$. But f is almost contra $R - \mathcal{I}-$continuous. So $f^{-1}(U)$ and $f^{-1}(V)$ are $R - \mathcal{I}-$open sets in X containing x and y respectively. Also $x \notin f^{-1}(V)$ and $y \notin f^{-1}(U)$. Hence X is $R - \mathcal{I} - T_1$.

Remark 4.1. If $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is a contra $R - \mathcal{I}-$continuous and one-one function where Y is weakly Hausdorff, then X is $R - \mathcal{I} - T_1$.

Theorem 4.6. *If $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is an almost contra $R - \mathcal{I}-$continuous and onto function where X is $R - \mathcal{I}-$connected, then Y is connected.*

Proof.
On the contrary assume that Y is disconnected. Then $Y = A \cup B$, where A and B are disjoint non-empty clopen subsets of Y. A and B are regular open in Y since they are clopen. Since f is almost contra $R - \mathcal{I}-$continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are $R - \mathcal{I}-$open in X. Since f is onto, $f^{-1}(A) \cup f^{-1}(B) = f^{-1}(A \cup B) = X$ and $f^{-1}(A) \cap f^{-1}(B) = f^{-1}(A \cap B) = \phi$ which contradicts to the fact that X is $R - \mathcal{I}-$connected. Hence Y is connected.

Theorem 4.7. *Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be a surjective almost contra-$R - \mathcal{I}-$continuous function. Then the following statements are true:*

1. *X is $R - \mathcal{I}-$compact implies Y is S-closed*
2. *X is $R - \mathcal{I}-$Lindeloff implies Y is S-Lindeloff*
3. *X is countably $R - \mathcal{I}-$compact implies Y is countably S-closed*

Proof.
1. Let $\{V_j : j \in J\}$ be an regular closed cover of Y. Then $\{f^{-1}(V_j) : j \in J\}$ is a $R - \mathcal{I}-$open cover of X since f is almost contra-$R - \mathcal{I}-$continuous. But X is $R - \mathcal{I}-$compact. Hence there exists a finite subset J_κ of J such that $X = \cup_{j \in J_\kappa}\{(f^{-1}(V_j))\}$. So $Y = \cup\{V_j : j \in J_\kappa\}$ and thus Y is S-closed.
2. and 3. can be proved in a similar fashion.

Theorem 4.8. *Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be a surjective almost contra-$R - \mathcal{I}-$continuous function. Then the following statements are true:*

1. *X is R-I- closed compact implies Y is nearly compact*
2. *X is $R - \mathcal{I}-$closed Lindeloff implies Y is nearly Lindeloff*
3. *X is countably $R - \mathcal{I}-$closed compact implies Y is nearly countably compact*

Proof.
1. Let $\{V_j : j \in J\}$ be an regular open cover of Y. Then $\{f^{-1}(V_j) : j \in J\}$ is a $R-\mathcal{I}-$closed cover of X since f is almost contra-$R - \mathcal{I}-$continuous. But X is $R - \mathcal{I}-$closed compact. Hence there exists a finite subset J_κ of J such that $X = \cup_{j \in J_\kappa}\{(f^{-1}(V_j))\}$. So $Y = \cup\{V_j : j \in J_\kappa\}$ and thus Y is nearly compact.
2. and 3. can be proved in a similar fashion.

5 Graphs

Definition 5.1. For a function $f : X \to Y$, the subset $\{(x, f(x)) : x \in X\} \subset X \times Y$ is called the graph of f and is denoted by $G(f)$.

Definition 5.2. The graph $G(f)$ of a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is said to be $R - \mathcal{I}-$closed (resp. contra $R - \mathcal{I}-$closed) if for each $(x, y) \in X \times Y \setminus G(f)$, there exist a $R - \mathcal{I}-$open set $U \in X$ containing x and an open (resp. a closed) set $V \in Y$ containing y such that $(U \times V) \cap G(f) = \phi$.

Definition 5.3. The graph $G(f)$ of a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is said to be strongly contra $R - \mathcal{I}-$closed if for each $(x, y) \in X \times Y \setminus G(f)$, there exist a $R - \mathcal{I}-$open set $U \in X$ containing x and a regular closed set $V \in Y$ containing y such that $(U \times V) \cap G(f) = \phi$.

Definition 5.4. (Sangeetha, M. V., & Chacko, B. 2018) A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is called $R - \mathcal{I}-$dense if $R - \mathcal{I} - Cl(A) = X$.

Lemma 5.1. *The graph $G(f)$ of a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is said to be $R - \mathcal{I}-$closed (resp. contra $R - \mathcal{I}-$closed) if and only if for each $(x, y) \in X \times Y \setminus G(f)$, there exist a $R - \mathcal{I}-$open set $U \in X$ containing x and an open (resp. a closed) set $V \in Y$ containing y such that $f(U) \cap V = \phi$.*

Proof.
Suppose that $(U \times V) \cap G(f) \neq \phi$. Then there exists at least one $(x, y) \in X \times Y$ such that $(x, y) \in U \times V$ and $(x, y) \in G(f)$. That means, $x \in U$, $y \in V$ and $V \ni y = f(x) \in f(U)$. Hence $f(U) \cap V \neq \phi$.

Lemma 5.2. *The graph $G(f)$ of a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is said to be strongly contra $R - \mathcal{I}-$closed if and only if for each $(x, y) \in X \times Y \setminus G(f)$, there exist a $R-\mathcal{I}-$open set $U \in X$ containing x and a regular closed set $V \in Y$ containing y such that $f(U) \cap V = \phi$.*

Theorem 5.3. *If a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is contra $R-\mathcal{I}-$continuous and Y is Urysohn, then $G(f)$ is contra $R - \mathcal{I}-$closed in $X \times Y$.*

Proof.
Let $(x, y) \in X \times Y \setminus G(f)$. So $f(x) \neq y$. Y is Urysohn implies that there exist open sets P and Q of Y containing $f(x)$ and y respectively such that $Cl(P) \cap Cl(Q) = \phi$. Since $Cl(P)$ is a closed set containing $f(x)$ and since f is contra $R - \mathcal{I}-$continuous, there exist a $R - \mathcal{I}-$open set $U \in X$ containing x such that $f(U) \subset Cl(P)$. Hence $f(U) \cap Cl(Q) = \phi$ and so $G(f)$ is contra $R-\mathcal{I}-$closed in $X \times Y$ by lemma 4.

Theorem 5.4. *If a function $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma)$ is $R-\mathcal{I}-$continuous and Y is T_1, then $G(f)$ is contra $R-\mathcal{I}-$closed in $X \times Y$.*

Proof.
Let $(x, y) \in X \times Y \setminus G(f)$. So $f(x) \neq y$. Since Y is T_1 there exists an open set $V \in Y$ such that $f(x) \in V,\ y \notin V$. Since f is $R-\mathcal{I}-$continuous, there exists an $R-\mathcal{I}-$open set $U \in X$ containing x such that $f(U) \subset V$. Then $f(U) \cap (Y \setminus V) = \phi$. But $(Y \setminus V)$ is a closed set in Y containing y. Thus $G(f)$ is contra $R-\mathcal{I}-$closed in $X \times Y$ by lemma 4.

Theorem 5.5. *Let Y be a Urysohn space. If $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma)$ and $g : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma)$ are contra $R-\mathcal{I}-$continuous functions, then $S = \{x \in X : f(x) = g(x)\}$ is $R-\mathcal{I}-$closed in X.*

Proof.
Let $x \notin S$. That is $x \in (X \setminus S)$ and so $f(x) \neq g(x)$. Since Y is Urysohn, there exist open sets P and Q of Y containing $f(x)$ and $g(x)$ respectively such that $Cl(P) \cap Cl(Q) = \phi$. Since f and g is contra $R-\mathcal{I}-$continuous, $f^{-1}(Cl(P))$ and $g^{-1}(Cl(Q))$ are $R-\mathcal{I}-$open in X containing x. Name $f^{-1}(Cl(P)) = A$ and $g^{-1}(Cl(Q)) = B$ and let $R = A \cap B$. Then R is a $R-\mathcal{I}-$open set in X containing x. Now, $f(R) \cap g(R) = f(A \cap B) \cap g(A \cap B) \subset f(A) \cap g(B) = Cl(P) \cap Cl(Q) = \phi$. Hence $S \cap R = \phi \longrightarrow S \subset (X \setminus R) \longrightarrow R-\mathcal{I}-Cl(S) \subset (X \setminus R) \longrightarrow x \notin R-\mathcal{I}-Cl(S)$. Thus $S = \{x \in X : f(x) = g(x)\}$ is $R-\mathcal{I}-$closed in X.

Theorem 5.6. *Let Y be a Urysohn space. If $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma)$ almost weakly $R-\mathcal{I}-$continuous, then $G(f)$ is strongly contra $R-\mathcal{I}-$closed in $X \times Y$.*

Proof.
Let $(x, y) \in X \times Y \setminus G(f)$. So $f(x) \neq y$. Y is Urysohn implies that there exist open sets P and Q of Y containing $f(x)$ and y respectively such that $Cl(P) \cap Cl(Q) = \phi$. Since f is almost weakly $R-\mathcal{I}-$continuous, there exists an $R-\mathcal{I}-$open set $U \in X$ containing x such that $f(U) \subset Cl(P)$. Hence $f(U) \cap Cl(Q) = \phi \longrightarrow f(U) \cap Cl(Int(Q))\phi$. Since $Cl(Int(Q))$ is regular closed in Y, $G(f)$ is strongly contra $R-\mathcal{I}-$closed in $X \times Y$ by lemma 5.

Theorem 5.7. *If a function $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma)$ is almost $R-\mathcal{I}-$continuous and Y is T_2, then $G(f)$ is strongly contra $R-\mathcal{I}-$closed in $X \times Y$.*

Proof.
Let $(x, y) \in X \times Y \setminus G(f)$. So $f(x) \neq y$. Since Y is T_2 there exist disjoint open sets A and B containing $f(x)$ and y respectively. Clearly $Cl(B)$ is regular closed and $Int(Cl(A))$ is regular open. A and B are disjoint implies $Cl(B) \cap Int(Cl(A)) = \phi$. Since f is almost $R-\mathcal{I}-$continuous, by lemma 3, there exists a $R-\mathcal{I}-$open set $U \in X$ containing x such that $f(U) \subset Int(Cl(A))$. Hence $f(U) \cap Cl(B) = \phi$ and thus $G(f)$ is strongly contra $R-\mathcal{I}-$closed in $X \times Y$ by lemma 5.

Theorem 5.8. *Let Y be a Urysohn space and $D \subset X$ be a $R-\mathcal{I}-$dense set. Let $f,\ g : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma)$ be any two contra $R-\mathcal{I}-$continuous functions. If $f = g$ on D, then $f = g$ on X.*

Proof.
By theorem 5.5, $S = \{x \in X : f(x) = g(x)\}$ is $R-\mathcal{I}-$closed in X. Clearly $D \subset S$. Since D is $R-\mathcal{I}-$dense, $R-\mathcal{I}-Cl(D) = X$. Thus $X = R-\mathcal{I}-Cl(D) \subset R-\mathcal{I}-Cl(S) = S$. Hence $f = g$ on X.

6 Conclusion

In this study, we introduced and explored the properties of contra $R-\mathcal{I}-$continuous and almost contra $R-\mathcal{I}-$continuous functions. We provided descriptions within the context of ideal topological spaces. We looked at how these functions behave under different topological conditions and examined the structural properties of contra $R-\mathcal{I}-$closed graphs.

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