Original Research Article

Some results on Generalized Fractional Calculus Operators Associated with the Product of the general class of Polynomials and extended Mittag-Leffler type Function

Abstract

The main objective of this paper is to evaluate four image formulas using generalized fractional calculus operators, specifically covering both fractional integration and fractional differentiation operators. In this study, these operators are applied to the product of the general class of polynomials (also known as Srivastava polynomials) and the extended Mittag-Leffler type function. The analytical results derived in this work are explicitly expressed in terms of the generalized Wright hypergeometric function. Because of the general nature and flexibility of the extended Mittag-Leffler function, the general class of polynomials, and the Wright hypergeometric function, many existing formulas in the literature as well as several new results can be easily obtained as special cases of our main findings. The theoretical framework established here provides useful tools for applications in fractional differential equations, applied mathematics, and mathematical physics.

Keywords and phrases: Generalized fractional calculus operators, extended Mittag-Leffler type function, generalized Wright hypergeometric function, Srivastava polynomials and special functions.

Mathematics subject classification: 26A33, 33B15, 33C45, 33C70, 33E12.

1 Introduction

In 1903, the Swedish mathematician Gosta Mittag-Leffler [14] studied the function $E_\mu(z)$ and defined as

$$E_\mu(z)=\sum_{r=0}^{\infty}\frac{z^r}{\Gamma(\mu r+1)},\left(\mu,z\in C,\Re(\mu)>0\right), \tag{1.1}$$

the Mittag-Leffler function (1.1) is a generalization of $exp(z)$ for $\mu=1$.

A generalization of the function $E_\mu(z)$ was introduced by Wiman [30] and expressed as

$$E_{\mu,\upsilon}(z)=\sum_{r=0}^{\infty}\frac{z^r}{\Gamma(\mu r+\upsilon)},\left(\mu,\upsilon,z\in C,\Re(\mu)>0,\Re(\upsilon)>0\right). \tag{1.2}$$

Further, in 1971 the new generalization of Mittag-Leffler function was studied by Prabhakar [17] in terms of the sereis representation as

$$E_{\mu,\upsilon}^{\lambda}(z)=\sum_{r=0}^{\infty}\frac{(\lambda)_r z^r}{\Gamma(\mu r+\upsilon)r!},\ (\mu,\upsilon,\lambda,z\in C,\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>0) \tag{1.3}$$

where $(\lambda)_r=\frac{\Gamma(\lambda+r)}{\Gamma(\lambda)}$ denotes the Pochhammer symbol (see [5], [7]).

In 2015, Parmar [16] defined the extended Mittag-Leffler type function in the following manner as (see also [2, 3, 24])

$$\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(z;p)=\sum_{r=0}^{\infty}\frac{\mathrm{B}^{\left(\{k_\ell\}_{\ell\in N_0}\right)}(\lambda+r,1-\lambda;p)}{\mathrm{B}(\lambda,1-\lambda)}\times\frac{z^r}{\Gamma(\mu r+\upsilon)}, \tag{1.4}$$

where $z,\upsilon,\lambda\in C;\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>1;\ p\geq 0$ and $\mathrm{B}^{\left(\{k_\ell\}_{\ell\in N_0}\right)}(\alpha,\beta;p)$ be the extended beta function defined by Srivastava et al. [25] in the integral form as follows:

$$\mathrm{B}^{\left(\{k_\ell\}_{\ell\in N_0}\right)}(\alpha,\beta;p)=\int_0^1 z^{\alpha-1}(1-z)^{\beta-1}\times\Theta\left(\{k_\ell\}_{\ell\in N_0};\frac{-p}{z(1-z)}\right)dz, \tag{1.5}$$

where $min\{\Re(\alpha),\Re(\beta)\}>0;\ p\geq 0$ and $\Theta\left(\{k_\ell\}_{\ell\in N_0};y\right)$ be a function of an appropriately bounded sequence $\{k_\ell\}_{\ell\in N_0}$ of arbitrary numbers (real or complex) defined as ([see [25], p-243, eq.(2.1))

$$\Theta\left(\{k_\ell\}_{\ell\in N_0};y\right)=\begin{cases}\sum_{\ell=0}^{\infty}k_\ell\frac{y^\ell}{\ell!} & \left(|y|<r;0<r<\infty;k_0=1\right)\\ \mathrm{M}_0\,y^{\rho}\exp(y)\left[1+O\left(\frac{1}{y}\right)\right] & \left(\Re(y)\to\infty,\mathrm{M}_0>0,\rho\in C\right)\end{cases}. \tag{1.6}$$

Here, 'r' be the radius of convergence for some suitable constants M_0 *and* ρ, depending on the sequence $\{k_\ell\}_{\ell\in N_0}$.

Remark 1: If we take $k_\ell=\frac{(\rho)_\ell}{(\sigma)_\ell}(\ell\in N_0)$ in (1.4), then $\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(z;p)$ reduce to another form of extended Mittag-Leffler function $\mathrm{E}_{\mu,\nu}^{(\rho,\sigma);\lambda}(z;p)$ (see [16], p-1072, eq. (17)) as (see also [2, 3, 24])

$$\mathrm{E}_{\mu,\nu}^{(\rho,\sigma);\lambda}(z;p)=\sum_{r=0}^{\infty}\frac{\mathrm{B}^{(\rho,\sigma)}(\lambda+r,1-\lambda;p)}{\mathrm{B}(\lambda,1-\lambda)}\times\frac{z^r}{\Gamma(\mu r+\upsilon)}, \tag{1.7}$$

$(z,\upsilon,\lambda\in C;\Re(\rho)>0,\Re(\sigma)>0,\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>1;\ p\geq 0)$.

Remark 2: If we set $k_\ell=1(\ell\in\mathrm{N}_0)$ in (1.4), then $\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(z;p)$ reduce to

$\mathrm{E}_{\mu,\upsilon}^{\lambda}(z;p)$ given by *Özarslan* and Yilmaz [15] as (see also [2, 3, 24])

$$\mathrm{E}_{\mu,\upsilon}^{\lambda}(z;p)=\sum_{r=0}^{\infty}\frac{\mathrm{B}(\lambda+r,1-\lambda;p)}{\mathrm{B}(\lambda,1-\lambda)}\times\frac{z^r}{\Gamma(\mu r+\upsilon)}, \tag{1.8}$$

$$(z,\upsilon,\lambda\in C;\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>1;p\geq 0).$$

Remark 3: If we take $k_\ell=1(\ell\in N_0)\,and\ p=0$, in (1.4), then $\mathrm{E}_{\mu,\upsilon}^{(\{k_\ell\}_{\ell\in N_0};\lambda)}(z;p)$ reduce to the Prabhakar type Mittag-Leffler function $E_{\mu,\upsilon}^{\lambda}(z)$ (see [6, 11, 22]).

Remark 4: If we put $\lambda=1$ and $\lambda=\upsilon=1$ in Prabhakar type Mittag-Leffler function $\mathrm{E}_{\mu,\upsilon}^{\lambda}(z;p)$, then it reduces to Wiman type Mittag-Leffler function $\mathrm{E}_{\mu,\upsilon}^{1}(z)=\mathrm{E}_{\mu,\upsilon}(z)$ and Gosta Mittag-Leffler function $\mathrm{E}_{\mu,1}^{1}(z)=\mathrm{E}_{\mu}(z)$ respectively (see [6, 11, 22]).

In 2018, Agarwal et al.[1] introduced an useful extension of the Wright type hypergeometric function in the following manner as

$${}_{m+1}\psi_{n+1}^{(\{k_\ell\}_{\ell\in N_0})}(\xi;p)={}_{m+1}\psi_{n+1}^{(\{k_\ell\}_{\ell\in N_0})}\left[\begin{matrix}(a_i,\mathrm{A}_i)_{1,m},(\lambda,1)\\(b_j,\mathrm{B}_j)_{1,n},(d,1)\end{matrix};(\xi;p)\right]$$

$$=\frac{1}{\Gamma(d-\lambda)}\sum_{r=0}^{\infty}\frac{\prod_{i=1}^{m}\Gamma(a_i+\mathrm{A}_i r)}{\prod_{j=1}^{n}\Gamma(b_j+\mathrm{B}_j r)}\mathrm{B}^{(\{k_\ell\}_{\ell\in N_0})}(\lambda+r,d-\lambda;p)\times\frac{\xi^r}{r!}, \tag{1.9}$$

where $\xi,\lambda\in C,\Re(d)>\Re(\lambda);p\geq 0.$

In 2015, Sharma and Devi [23] extended the Wright type hypergeometric function as follows

$${}_{m+1}\psi_{n+1}(\xi;p)={}_{m+1}\psi_{n+1}\left[\begin{matrix}(a_i,\mathrm{A}_i)_{1,m},(\lambda,1)\\(b_j,\mathrm{B}_j)_{1,n},(d,1)\end{matrix};(\xi;p)\right]$$

$$=\frac{1}{\Gamma(d-\lambda)}\sum_{r=0}^{\infty}\frac{\prod_{i=1}^{m}\Gamma(a_i+\mathrm{A}_i r)}{\prod_{j=1}^{n}\Gamma(b_j+\mathrm{B}_j r)}\mathrm{B}_p(\lambda+r,d-\lambda)\times\frac{\xi^r}{r!}, \tag{1.10}$$

where $\xi,\lambda\in C,\Re(p)>0,\Re(d)>\Re(\lambda)>0$ and $B_p(x,y)$ be the extended beta function (see [23], p-46, eq. (3)).

The generalized Wright hypergeometric function was defined by Wright [31] in the following series as

$$ {}_m\psi_n(\xi) = {}_m\psi_n\left[\begin{matrix}(a_i, A_i)_{1,m} \\ (b_j, B_j)_{1,n}\end{matrix};\xi\right] = \sum_{r=0}^{\infty} \frac{\prod_{i=1}^{m}\Gamma(a_i + A_i r)}{\prod_{j=1}^{n}\Gamma(b_j + B_j r)} \frac{\xi^r}{r!}, \tag{1.11} $$

where $\xi, a_i, b_j \in C\ and\ A_i, B_j \in R - \{0\}, (i = 1,\dots,m; j = 1,\dots,n)$ with condition $\sum_{j=1}^{q} B_j - \sum_{i=1}^{p} A_i > -1.$

Remark 5: If we take $k_\ell = 1(\ell \in N_0)$, then (1.9) reduce to (1.10); further, if we put $p = 0$, then (1.9) reduce to (1.11). (see [2, 3])

The general class of polynomials (also known as Srivastava polynomials) $S_n^m[\xi]$ defined by Srivastava [26, 27] as follows: (see also [8, 12, 22])

$$ S_n^m[\xi] = \sum_{s=0}^{[n/m]} \frac{(-n)_{ms}}{s!} \quad A_{n,s}\ \xi^s, \qquad (n = 0,1,2,...), \tag{1.12} $$

where m is an arbitrary positive integer and the coefficients $A_{n,s}(n, s \geq 0)$ are arbitrary constants, real or complex. On suitably specializing the coefficients $A_{n,s}$, the polynomial family $S_n^m[\xi]$ yields a number of known polynomials as its special cases. These include, among others, the Hermite polynomials, the Jacobi polynomials, the Laguerre polynomials, the Bessel's polynomials and several others (see [28, 29]).

2 Generalized fractional calculus operators

Let $\alpha, \alpha', \eta, \eta', \delta \in C, x > 0$ and $\Re(\delta) > 0,$ the generalized fractional integral and differential operators involving Appell's function (also known as Horn's function) $F_3(.)$ are defined by Saigo and Maeda (see [19], p-393, eq.(4.12) and (4.13)) as follows: (see also [12, 13, 20, 21, 22])

$$ \left(I_{0+}^{\alpha,\alpha',\eta,\eta',\delta} f\right)(x) = \frac{x^{-\alpha}}{\Gamma(\delta)} \int_0^x (x-t)^{\delta-1} t^{-\alpha'} F_3\left(\alpha, \alpha', \eta, \eta'; \delta; 1 - \frac{t}{x}, 1 - \frac{x}{t}\right) f(t)\, dt, \tag{2.1} $$

$$ \left(I_{-}^{\alpha,\alpha',\eta,\eta',\delta} f\right)(x) = \frac{x^{-\alpha'}}{\Gamma(\delta)} \int_x^\infty (t-x)^{\delta-1} t^{-\alpha} F_3\left(\alpha, \alpha', \eta, \eta'; \delta; 1 - \frac{x}{t}, 1 - \frac{t}{x}\right) f(t)\, dt, \tag{2.2} $$

$$ \left(D_{0+}^{\alpha,\alpha',\eta,\eta',\delta} f\right)(x) = \left(I_{0+}^{-\alpha',-\alpha,-\eta',-\eta,-\delta} f\right)(x) \tag{2.3} $$

$$ = \left(\frac{d}{dx}\right)^k \left(I_{0+}^{-\alpha',-\alpha,-\eta'+k,-\eta,-\delta+k} f\right)(x), \left(\Re(\delta) > 0; k = \left[\Re(\delta) + 1\right]\right) \tag{2.4} $$

and

$$\left(D_{-}^{\alpha,\alpha',\eta,\eta',\delta} f\right)(x) = \left(I_{-}^{-\alpha',-\alpha,-\eta',-\eta,-\delta} f\right)(x) \tag{2.5}$$

$$= \left(\frac{-d}{dx}\right)^{k} \left(I_{-}^{-\alpha',-\alpha,-\eta',-\eta+k,-\delta+k} f\right)(x), \left(\Re(\delta) > 0; k = \left[\Re(\delta)+1\right]\right) \tag{2.6}$$

Let $\alpha,\eta,\delta \in C$ and $x > 0$, then the Saigo fractional calculus operators [18] associated with Gauss hypergeometric function ${}_2F_1(.)$ are defined for $\Re(\alpha) > 0$, as follows:

$$\left(I_{0+}^{\alpha,\eta,\delta} f\right)(x) = \frac{x^{-\alpha-\eta}}{\Gamma(\delta)} \int_0^x (x-t)^{\alpha-1} \,{}_2F_1\left(\alpha+\eta,-\delta;\alpha;1-\frac{t}{x}\right) f(t)\,dt, \tag{2.7}$$

$$\left(I_{-}^{\alpha,\eta,\delta} f\right)(x) = \frac{1}{\Gamma(\delta)} \int_x^\infty (t-x)^{\alpha-1}\, t^{-\alpha-\eta} \,{}_2F_1\left(\alpha+\eta,-\delta;\alpha;1-\frac{x}{t}\right) f(t)\,dt, \tag{2.8}$$

$$\left(D_{0+}^{\alpha,\eta,\delta} f\right)(x) = \left(I_{0+}^{-\alpha,-\eta,\alpha+\delta} f\right)(x) = \left(\frac{d}{dx}\right)^{k} \left(I_{0+}^{-\alpha+k,-\eta-k,\alpha+\delta-k} f\right)(x), (k = \left[\Re(\alpha)+1\right]), \tag{2.9}$$

and

$$\left(D_{-}^{\alpha,\eta,\delta} f\right)(x) = \left(I_{-}^{-\alpha,-\eta,\alpha+\delta} f\right)(x) = \left(\frac{-d}{dx}\right)^{k} \left(I_{-}^{-\alpha+k,-\eta-k,\alpha+\delta} f\right)(x), (k = \left[\Re(\alpha)+1\right]). \tag{2.10}$$

Remark 6: If we take $\alpha = \alpha+\eta, \alpha' = \eta' = 0, \eta = -\delta$ and $\delta = \alpha$, operators given in (2.1)-(2.6) reduce to the Saigo fractional calculus operators as given by (2.7)-(2.10). (see [4, 9, 10])
Remark 7: If we set $\eta = -\alpha$, then operators (2.7)-(2.10) reduce to Riemann-Liouville fractional calculus operators [,] as follows: (see [4, 9, 10])

$$\left(I_{0+}^{\alpha,-\alpha,\delta} f\right)(x) = \left(I_{0+}^{\alpha} f\right)(x), \tag{2.11}$$

$$\left(I_{-}^{\alpha,-\alpha,\delta} f\right)(x) = \left(I_{-}^{\alpha} f\right)(x), \tag{2.12}$$

$$\left(D_{0+}^{\alpha,-\alpha,\delta} f\right)(x) = \left(D_{0+}^{\alpha} f\right)(x), \tag{2.13}$$

and

$$\left(D_{-}^{\alpha,-\alpha,\delta} f\right)(x) = \left(D_{-}^{\alpha} f\right)(x), \tag{2.14}$$

Further, the following image formulas for a power function, under operators (2.1) and (2.2) are given by (see [5, 6, 21, 22])

$$\left(I_{0+}^{\alpha,\alpha',\eta,\eta',\delta} t^{\sigma-1}\right)(x) = x^{\sigma-\alpha-\alpha'+\delta-1}\, \Gamma\left[\begin{matrix} \sigma, \sigma+\delta-\alpha-\alpha'-\eta, \sigma+\eta'-\alpha' \\ \sigma+\delta-\alpha-\alpha', \sigma+\delta-\alpha'-\eta, \rho+\eta' \end{matrix}\right], \tag{2.15}$$

where $\Re(\delta) > 0, \Re(\sigma) > max\left[0, \Re(\alpha+\alpha'+\eta-\delta), \Re(\alpha'-\eta')\right]$,

and

$$\left(I_{-}^{\alpha,\alpha',\eta,\eta',\delta}t^{\sigma-1}\right)(x)=x^{\sigma-\alpha-\alpha'+\delta-1}\,\Gamma\left[\begin{array}{c}1+\alpha+\alpha'-\delta-\sigma,1+\alpha+\eta'-\delta-\sigma,1-\eta-\sigma\\1-\sigma,1+\alpha+\alpha'+\eta'-\delta-\sigma,1+\alpha-\eta-\sigma\end{array}\right], \tag{2.16}$$

where $\Re(\delta)>0, \Re(\sigma)<1+\min\left[\Re(-\eta), \Re(\alpha+\alpha'-\delta), \Re(\alpha+\eta'-\delta)\right]$.

Here, the Symbol $\Gamma\left[\begin{array}{c}\alpha,\beta,\gamma\\\mu,\upsilon,\lambda\end{array}\right]$ will be used to represent the ratio of product of gamma functions as $\dfrac{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma)}{\Gamma(\mu)\Gamma(\upsilon)\Gamma(\lambda)}$.

3 Main results

3.1 Generalized fractional integration of product of extended Mittag-Leffler function and Srivastava polynomials

In this section, we establish two image formulas for left-sided and right-sided generalized fractional integration involving the product of the general class of polinomials (also known as Srivastava polynomials) and extended Mittag-Leffler type function in terms of the generalized Wright hypergeometric function. These formulas are given by the following theorems:

Theorem 1 Let $\alpha,\alpha',\eta,\eta',\delta,\mu,\upsilon,\lambda,p\in C, \Re(\mu)>0, \Re(\upsilon)>0, \Re(\lambda)>1, p\geq 0, t,x>0$ and $a,\omega\in R$ such that $\Re(\delta)>0$ and $\Re(\upsilon)>max\left\{0,\Re(\alpha+\alpha'+\eta-\delta),\Re(\alpha'-\eta')\right\}$. Then the left-sided generalized fractional integration $I_{0+}^{\alpha,\alpha',\eta,\eta',\delta}$ of the product of general class of polynomials $S_n^m[z]$ and extended Mittag-Leffler type function $\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(z;p)$ and is given by

$$\{I_{0+}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\upsilon-1}S_n^m\left[\omega t^\beta\right]\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(at;p)\right)\}(x)=\frac{x^{\upsilon+\delta-\alpha-\alpha'-1}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^\beta\right)^s$$

$$\times{}_4\psi_4^{\left(\{k_\ell\}_{\ell\in N_0}\right)}\left[\begin{array}{c}(\lambda,1),(\upsilon+\beta s,1),(\upsilon+\beta s+\delta-\alpha-\alpha'-\eta,1),(\upsilon+\beta s+\eta'-\alpha',1)\\(\upsilon,\mu),(\upsilon+\beta s+\delta-\alpha-\alpha',1),(\upsilon+\beta s+\delta-\alpha'-\eta,1),(\upsilon+\beta s+\eta',1)\end{array};(ax;p)\right] \tag{3.1}$$

Proof: By using series representation of extended Mittag-Leffler type function (1.4), the general class of polynomials (1.12) and left-sided fractional integration power function formula (2.15), the LHS of theorem leads to

$$\{I_{0+}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\upsilon-1}S_n^m\left[\omega t^\beta\right]\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(at;p)\right)\}(x)$$

$$=\left\{I_{0+}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\upsilon-1}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega t^\beta\right)^s\times\sum_{r=0}^{\infty}\frac{\mathrm{B}^{\left(\{k_\ell\}_{\ell\in N_0}\right)}(\lambda+r,1-\lambda;p)}{\mathrm{B}(\lambda,1-\lambda)}\times\frac{(at)^r}{\Gamma(\mu r+\upsilon)}\right)\right\}(x),$$

by interchanging the order of integration and summations, which is valid under the conditions of the theorem 1, we get

$$=\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\;\omega^{s}\times\sum_{r=0}^{\infty}\frac{\mathrm{B}^{\left(\{k_\ell\}_{\ell\in N_0}\right)}(\lambda+r,1-\lambda;p)}{\mathrm{B}(\lambda,1-\lambda)}\times\frac{a^{r}}{\Gamma(\mu r+\upsilon)}\left(I_{0+}^{\alpha,\alpha',\eta,\eta',\delta}t^{\upsilon+\beta s+r-1}\right)(x)$$

$$=\frac{x^{\upsilon+\delta-\alpha-\alpha'-1}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^{s}\times\sum_{r=0}^{\infty}\frac{\mathrm{B}^{\left(\{k_\ell\}_{\ell\in N_0}\right)}(\lambda+r,1-\lambda;p)}{\mathrm{B}(\lambda,1-\lambda)}$$

$$\times\frac{\Gamma(\upsilon+\beta s+r)\Gamma(\upsilon+\beta s+\delta-\alpha-\alpha'-\eta+r)\Gamma(\upsilon+\beta s+\eta'-\alpha'+r)}{\Gamma(\upsilon+\beta s+\delta-\alpha-\alpha'+r)\Gamma(\upsilon+\beta s+\delta-\alpha'-\eta+r)\Gamma(\upsilon+\beta s+\eta'+r)}\times\frac{(ax)^{r}}{\Gamma(\mu r+\upsilon)},\qquad(3.2)$$

next, by interpreting the terms of the above equation (3.2), in view of definition(1.9), we arrive at the result (3.1), which completes the proof of theorem 1.

On setting $n=0, A_{0,0}=1$ then $S_0^m[z]\to 1$ in theorem 1, we obtain the following result as:

Corollary 1.1 Let $\alpha,\alpha',\eta,\eta',\delta,\mu,\upsilon,\lambda,p\in C,\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>1,p\geq 0,t,x>0$ and $a,\omega\in R$ such that $\Re(\delta)>0$ and $\Re(\upsilon)>max\{0,\Re(\alpha+\alpha'+\eta-\delta),\Re(\alpha'-\eta')\}$. Then there holds the formula:

$$\{I_{0+}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\upsilon-1}\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(at;p)\right)\}(x)$$

$$=\frac{x^{\upsilon+\delta-\alpha-\alpha'-1}}{\Gamma(\lambda)}\times{}_4\psi_4^{\left(\{k_\ell\}_{\ell\in N_0}\right)}\left[\begin{matrix}(\lambda,1),(\upsilon,1),(\upsilon+\delta-\alpha-\alpha'-\eta,1),(\upsilon+\eta'-\alpha',1)\\(\upsilon,\mu),(\upsilon+\delta-\alpha-\alpha',1),(\upsilon+\delta-\alpha'-\eta,1),(\upsilon+\eta',1)\end{matrix};(ax;p)\right]\qquad(3.3)$$

on taking $\alpha=\alpha+\eta,\alpha'=\eta'=0,\eta=-\delta$ and $\delta=\alpha$, Theorem 1 reduces to the following corollary:

Corollary 1.2 Let $\alpha,\beta,\eta,\mu,\upsilon,\lambda,p\in C,\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>1,p\geq 0,t,x>0$ and $a,\omega\in R$ such that $\Re(\alpha)>0$ and $\Re(\upsilon)>max\{0,\Re(\eta-\delta)\}$. Then the following integral formula holds:

$$\{I_{0+}^{\alpha,\eta,\delta}\left(t^{\upsilon-1}S_n^m\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(at;p)\right)\}(x)=\frac{x^{\upsilon-\eta-1}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^{s}$$

$$\times{}_3\psi_3^{\left(\{k_\ell\}_{\ell\in N_0}\right)}\left[\begin{matrix}(\lambda,1),(\upsilon+\beta s,1),(\upsilon+\beta s+\delta-\eta,1)\\(\upsilon,\mu),(\upsilon+\beta s-\eta,1),(\upsilon+\beta s+\delta+\alpha,1)\end{matrix};(ax;p)\right]\qquad(3.4)$$

Now, on putting $\eta=-\alpha$ in corollary (1.2), we get the following result as:

Corollary 1.3 Let $\alpha,\mu,\upsilon,\lambda,p\in C,\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>1,p\geq 0,t,x>0$ and $a,\ \omega\in R$ such that $\Re(\alpha)>0$ Then there holds the following formula:

$$\{I_{0+}^{\alpha}\left(t^{\upsilon-1}S_n^m\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}\left(at;p\right)\right)\}\left(x\right)$$

$$=\frac{x^{\upsilon+\alpha-1}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^s\times{}_2\psi_2^{\left(\{k_\ell\}_{\ell\in N_0}\right)}\left[\begin{matrix}(\lambda,1),(\upsilon+\beta s,1)\\(\upsilon,\mu),(\upsilon+\beta s+\alpha,1)\end{matrix};(ax;p)\right] \tag{3.5}$$

Next, if we put $k_\ell=\frac{(\rho)_\ell}{(\sigma)_\ell}\left(\ell\in N_0\right)$ in (3.1), then $\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(z;p)$ reduce to $\mathrm{E}_{\mu,\upsilon}^{(\rho,\sigma);\lambda}\left(z;p\right)$, we arrive at the following result as

Corollary 1.4 Let $\alpha,\alpha',\eta,\eta',\delta,\rho,\sigma,\mu,\upsilon,\lambda,p\in C,\Re\left(\rho\right)>0,\Re\left(\sigma\right)>0,\Re\left(\mu\right)>0,\Re\left(\upsilon\right)>0,$ $\Re\left(\lambda\right)>1,p\geq 0,t,x>0$ and $a,\ \omega\in R$ such that $\Re\left(\delta\right)>0$ and $\Re\left(\upsilon\right)>max\left\{0,\Re\left(\alpha+\alpha'+\eta-\delta\right),\Re\left(\alpha'-\eta'\right)\right\}$. Then there holds the following formula:

$$\{I_{0+}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\upsilon-1}S_n^m\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{(\rho,\sigma);\lambda}\left(at;p\right)\right)\}\left(x\right)=\frac{x^{\upsilon+\delta-\alpha-\alpha'-1}\Gamma\left(\sigma\right)}{\Gamma(\lambda)\Gamma\left(\rho\right)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^s$$

$$\times{}_5\psi_5\left[\begin{matrix}(\lambda,1),(\upsilon+\beta s,1),(\upsilon+\beta s+\delta-\alpha-\alpha'-\eta,1),(\upsilon+\beta s+\eta'-\alpha',1),(\rho,1)\\(\upsilon,\mu),(\upsilon+\beta s+\delta-\alpha-\alpha',1),(\upsilon+\beta s+\delta-\alpha'-\eta,1),(\upsilon+\beta s+\eta',1),(\sigma,1)\end{matrix};(ax;p)\right] \tag{3.6}$$

Again, if we set $k_\ell=1\left(\ell\in N_0\right)$ in (3.1),then $\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(z;p)$ reduce to $\mathrm{E}_{\mu,\nu}^{\lambda}\left(z;p\right)$, we get the following corollary as

Corollary 1.5 Let $\alpha,\alpha',\eta,\eta',\delta,\mu,\upsilon,\lambda,p\in C,\Re\left(\mu\right)>0,\Re\left(\upsilon\right)>0,\Re\left(\lambda\right)>1,p\geq 0,t,x>0$ and $a,\ \omega\in R$ such that $\Re\left(\delta\right)>0$ and $\Re\left(\upsilon\right)>max\left\{0,\Re\left(\alpha+\alpha'+\eta-\delta\right),\Re\left(\alpha'-\eta'\right)\right\}$. Then the following result holds:

$$\{I_{0+}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\upsilon-1}S_n^m\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{\lambda}\left(at;p\right)\right)\}\left(x\right)=\frac{x^{\upsilon+\delta-\alpha-\alpha'-1}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^s$$

$$\times{}_4\psi_4\left[\begin{matrix}(\lambda,1),(\upsilon+\beta s,1),(\upsilon+\beta s+\delta-\alpha-\alpha'-\eta,1),(\upsilon+\beta s+\eta'-\alpha',1)\\(\upsilon,\mu),(\upsilon+\beta s+\delta-\alpha-\alpha',1),(\upsilon+\beta s+\delta-\alpha'-\eta,1),(\upsilon+\beta s+\eta',1)\end{matrix};(ax;p)\right] \tag{3.7}$$

Further, if we take $k_\ell=1\left(\ell\in N_0\right)and\ p=0$ in (3.1), then $\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(z;p)$ reduce to $\mathrm{E}_{\mu,\upsilon}^{\lambda}\left(z\right)$, then we obtain the following result as:

Corollary 1.6 Let $\alpha,\alpha',\eta,\eta',\delta,\mu,\upsilon,\lambda\in C,\Re\left(\mu\right)>0,\Re\left(\upsilon\right)>0,\Re\left(\lambda\right)>1,t,x>0$ and $a,\ \omega\in R$ such that $\Re\left(\delta\right)>0$ and $\Re\left(\upsilon\right)>max\left\{0,\Re\left(\alpha+\alpha'+\eta-\delta\right),\Re\left(\alpha'-\eta'\right)\right\}$. Then there holds the following formula true:

$$\{I_{0+}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\upsilon-1}S_n^m\left[\omega t^\beta\right]\mathrm{E}_{\mu,\upsilon}^{\lambda}\left(at\right)\right)\}\left(x\right)=\frac{x^{\upsilon+\delta-\alpha-\alpha'-1}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{\left(-n\right)_{ms}}{s!}A_{n,s}\left(\omega x^\beta\right)^s$$

$$\times{}_4\psi_4\left[\begin{matrix}(\lambda,1),(\upsilon+\beta s,1),(\upsilon+\beta s+\delta-\alpha-\alpha'-\eta,1),(\upsilon+\beta s+\eta'-\alpha',1)\\(\upsilon,\mu),(\upsilon+\beta s+\delta-\alpha-\alpha',1),(\upsilon+\beta s+\delta-\alpha'-\eta,1),(\upsilon+\beta s+\eta',1)\end{matrix};ax\right] \tag{3.8}$$

Theorem 2 Let $\alpha,\alpha',\eta,\eta',\delta,\mu,\upsilon,\lambda,p\in C,\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>1,p\geq 0,t,x>0,$ and $a,\omega\in R$ such that $\Re(\delta)>0$ and $\Re(1-\delta-\upsilon)<1+min\{\Re(-\eta),\Re(\alpha+\alpha'-\delta),\Re(\alpha+\eta'-\delta)\}$. Then the right-sided generalized fractional integration $I_{-}^{\alpha,\alpha',\eta,\eta',\delta}$ of the product of the general class polynomials $S_n^m[z]$ and extended Mittag-Leffler type function $\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(z;p)$ is given by

$$\{I_{-}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{-\upsilon-\delta}S_n^m\left[\omega t^\beta\right]\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}\left(at^{-1};p\right)\right)\}\left(x\right)=\frac{x^{-\upsilon-\alpha-\alpha'}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{\left(-n\right)_{ms}}{s!}A_{n,s}\left(\omega x^\beta\right)^s$$

$$\times{}_4\psi_4^{\left(\{k_\ell\}_{\ell\in N_0}\right)}\left[\begin{matrix}(\lambda,1),(\upsilon-\beta s+\alpha+\alpha',1),(\upsilon-\beta s+\alpha+\eta',1),(\upsilon-\beta s+\delta-\eta,1)\\(\upsilon,\mu),(\upsilon-\beta s+\delta,1),(\upsilon-\beta s+\alpha+\alpha'+\eta',1),(\upsilon-\beta s+\delta+\alpha-\eta,1)\end{matrix};\left(ax^{-1};p\right)\right] \tag{3.9}$$

Proof: By using series representation of extended Mittag-Leffler function (1.4), the general class of polynomials (1.12) and right-sided fractional integration power function formula (2.16), LHS of theorem 2, can be written as

$$\{I_{-}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{-\upsilon-\delta}S_n^m\left[\omega t^\beta\right]\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}\left(at^{-1};p\right)\right)\}\left(x\right)$$

$$=\left\{I_{-}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{-\upsilon-\delta}\sum_{s=0}^{[n/m]}\frac{\left(-n\right)_{ms}}{s!}A_{n,s}\left(\omega t^\beta\right)^s\times\sum_{r=0}^{\infty}\frac{\mathrm{B}^{\left(\{k_\ell\}_{\ell\in N_0}\right)}\left(\lambda+r,1-\lambda;p\right)}{\mathrm{B}\left(\lambda,1-\lambda\right)}\times\frac{\left(at^{-1}\right)^r}{\Gamma\left(\mu r+\upsilon\right)}\right)\right\}\left(x\right),$$

by interchanging the order of integration and summations, which is valid under the conditions of the theorem 2, we have

$$=\sum_{s=0}^{[n/m]}\frac{\left(-n\right)_{ms}}{s!}A_{n,s}\,\omega^s\times\sum_{r=0}^{\infty}\frac{\mathrm{B}^{\left(\{k_\ell\}_{\ell\in N_0}\right)}\left(\lambda+r,1-\lambda;p\right)}{\mathrm{B}\left(\lambda,1-\lambda\right)}\times\frac{a^r}{\Gamma\left(\mu r+\upsilon\right)}\left(I_{-}^{\alpha,\alpha',\eta,\eta',\delta}t^{1-\upsilon-\delta+\beta s-r-1}\right)\left(x\right)$$

$$=\frac{x^{-\upsilon-\alpha-\alpha'}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{\left(-n\right)_{ms}}{s!}A_{n,s}\left(\omega x^\beta\right)^s\times\sum_{r=0}^{\infty}\frac{\mathrm{B}^{\left(\{k_\ell\}_{\ell\in N_0}\right)}\left(\lambda+r,1-\lambda;p\right)}{\mathrm{B}\left(\lambda,1-\lambda\right)}$$

$$\times\frac{\Gamma(\upsilon-\beta s+\alpha+\alpha'+r)\Gamma(\upsilon-\beta s+\alpha+\eta'+r)\Gamma(\upsilon-\beta s+\delta-\eta+r)}{\Gamma(\upsilon-\beta s+\delta+r)\Gamma(\upsilon-\beta s+\alpha+\alpha'+\eta'+r)\Gamma(\upsilon-\beta s+\delta+\alpha-\eta+r)}\times\frac{(ax^{-1})^{r}}{\Gamma(\mu r+\upsilon)},\quad(3.10)$$

now, interpreting the terms of the above equation (3.10), in view of definition (1.9), we arrive at the result (3.9), which completes the proof of theorem 2.

On setting $n=0, A_{0,0}=1$ then $S_0^m[z]\to 1$ in (3.9), we obtain the following corollary as:

Corollary 2.1 Let $\alpha,\alpha',\eta,\eta',\delta,\mu,\upsilon,\lambda,p\in C,\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>1,p\geq 0,t,x>0,$ and $a,\ \omega\in R$ such that $\Re(\delta)>0$ and $\Re(1-\delta-\upsilon)<1+min\{\Re(-\eta),\Re(\alpha+\alpha'-\delta),\Re(\alpha+\eta'-\delta)\}$. Then there holds the formula:

$$\{I_{-}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{-\upsilon-\delta}\mathrm{E}_{\mu,\upsilon}^{(\{k_\ell\}_{\ell\in N_0};\lambda)}(at^{-1};p)\right)\}(x)$$

$$=\frac{x^{-\upsilon-\alpha-\alpha'}}{\Gamma(\lambda)}\times{}_4\psi_4^{(\{k_\ell\}_{\ell\in N_0})}\left[\begin{matrix}(\lambda,1),(\upsilon+\alpha+\alpha',1),(\upsilon+\alpha+\eta',1),(\upsilon+\delta-\eta,1)\\(\upsilon,\mu),(\upsilon+\delta,1),(\upsilon+\alpha+\alpha'+\eta',1),(\upsilon+\delta+\alpha-\eta,1)\end{matrix};(ax^{-1};p)\right]\quad(3.11)$$

If we take $\alpha=\alpha+\eta,\alpha'=\eta'=0,\eta=-\delta$ and $\delta=\alpha$, Theorem 2 reduces to the following corollary:

Corollary 2.2 Let $\alpha,\eta,\delta,\mu,\upsilon,\lambda,p\in C,\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>1,p\geq 0,t,x>0$ and $a,\ \omega\in R$ such that $\Re(\alpha)>0$ and $\Re(1-\delta-\upsilon)<1+min\{\Re(-\eta),\Re(-\delta)\}$. Then we have

$$\{I_{-}^{\alpha,\eta,\delta}\left(t^{-\upsilon-\alpha}S_n^m\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{(\{k_\ell\}_{\ell\in N_0};\lambda)}(at^{-1};p)\right)\}(x)=\frac{x^{-\upsilon-\alpha-\eta}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^{s}$$

$$\times{}_3\psi_3^{(\{k_\ell\}_{\ell\in N_0})}\left[\begin{matrix}(\lambda,1),(\upsilon-\beta s+\alpha+\eta,1),(\upsilon-\beta s+\delta+\alpha,1)\\(\upsilon,\mu),(\upsilon-\beta s+\alpha,1),(\upsilon-\beta s+\delta+2\alpha+\eta,1)\end{matrix};(ax^{-1};p)\right]\quad(3.12)$$

Further, if we put $\eta=-\alpha$ in corollary (2.2), we arrive at the following result:

Corollary 2.3 Le $\alpha,\mu,\upsilon,\lambda,p\in C,\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>1,p\geq 0,x>0$ and $a,\ \omega\in R$ such that $\Re(\alpha)>0$. Then the following result holds:

$$\{I_{-}^{\alpha}\left(t^{-\upsilon-\alpha}S_n^m\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{(\{k_\ell\}_{\ell\in N_0};\lambda)}(at^{-1};p)\right)\}(x)$$

$$=\frac{x^{-\upsilon}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^{s}\times{}_2\psi_2^{(\{k_\ell\}_{\ell\in N_0})}\left[\begin{matrix}(\lambda,1),(\upsilon-\beta s,1)\\(\upsilon,\mu),(\upsilon-\beta s+\alpha,1)\end{matrix};(ax^{-1};p)\right]\quad(3.13)$$

Next, if we put $k_\ell = \frac{(\rho)_\ell}{(\sigma)_\ell}(\ell \in N_0)$ in (3.9), then $E_{\mu,\upsilon}^{(\{k_\ell\}_{\ell\in N_0};\lambda)}(z;p)$ reduce to $E_{\mu,\upsilon}^{(\rho,\sigma);\lambda}(z;p)$, we arrive at the following result as:

Corollary 2.4 Let $\alpha,\alpha',\eta,\eta',\delta,\rho,\sigma\mu,\upsilon,\lambda,p \in C, \Re(\rho)>0, \Re(\sigma)>0, \Re(\mu)>0, \Re(\upsilon)>0,$ $\Re(\lambda)>1,\ p\geq 0,\ t,x>0,$ and $a,\ \omega \in R$ such that $\Re(\delta)>0$ and $\Re(1-\delta-\upsilon)<1+min\{\Re(-\eta),\Re(\alpha+\alpha'-\delta),\Re(\alpha+\eta'-\delta)\}$. Then there holds the following formula:

$$\{I_{-}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{-\upsilon-\delta}S_n^m\left[\omega t^\beta\right]E_{\mu,\upsilon}^{(\rho,\sigma);\lambda}\left(at^{-1};p\right)\right)\}(x)=\frac{x^{-\upsilon-\alpha-\alpha'}\Gamma(\sigma)}{\Gamma(\lambda)\Gamma(\rho)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^\beta\right)^s$$

$$\times{}_5\psi_5\left[\begin{matrix}(\lambda,1),(\upsilon-\beta s+\alpha+\alpha',1),(\upsilon-\beta s+\alpha+\eta',1),(\upsilon-\beta s+\delta-\eta,1),(\rho,1)\\(\upsilon,\mu),(\upsilon-\beta s+\delta,1),(\upsilon-\beta s+\alpha+\alpha'+\eta',1),(\upsilon-\beta s+\delta+\alpha-\eta,1),(\sigma,1)\end{matrix};\left(ax^{-1};p\right)\right] \tag{3.14}$$

Again, if we set $k_\ell = 1(\ell \in N_0)$ in (3.9), then $E_{\mu,\upsilon}^{(\{k_\ell\}_{\ell\in N_0};\lambda)}(z;p)$ reduce to $E_{\mu,\upsilon}^{\lambda}(z;p)$, we obtain the following result as

Corollary 2.5 Let $\alpha,\alpha',\eta,\eta',\delta,\mu,\upsilon,\lambda,p \in C, \Re(\mu)>0, \Re(\upsilon)>0, \Re(\lambda)>1, p\geq 0, t,x>0,$ and $a,\omega \in R$ such that $\Re(\delta)>0$ and $\Re(1-\delta-\nu)<1+min\{\Re(-\eta),\Re(\alpha+\alpha'-\delta),\Re(\alpha+\eta'-\delta)\}$. Then the following formula holds:

$$\{I_{-}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{-\upsilon-\delta}S_n^m\left[\omega t^\beta\right]E_{\mu,\upsilon}^{\lambda}\left(at^{-1};p\right)\right)\}(x)=\frac{x^{-\upsilon-\alpha-\alpha'}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^\beta\right)^s$$

$$\times{}_4\psi_4\left[\begin{matrix}(\lambda,1),(\upsilon-\beta s+\alpha+\alpha',1),(\upsilon-\beta s+\alpha+\eta',1),(\upsilon-\beta s+\delta-\eta,1)\\(\upsilon,\mu),(\upsilon-\beta s+\delta,1),(\upsilon-\beta s+\alpha+\alpha'+\eta',1),(\upsilon-\beta s+\delta+\alpha-\eta,1)\end{matrix};\left(ax^{-1};p\right)\right] \tag{3.15}$$

Further, if we take $k_\ell = 1(\ell \in N_0)$ *and* $p=0$ in (3.9), then $E_{\mu,\upsilon}^{(\{k_\ell\}_{\ell\in N_0};\lambda)}(z;p)$ reduce to $E_{\mu,\upsilon}^{\lambda}(z)$, then we obtain the following result as:

Corollary 2.6 Let $\alpha,\alpha',\eta,\eta',\delta,\mu,\upsilon,\lambda \in C, \Re(\mu)>0, \Re(\upsilon)>0, \Re(\lambda)>1, t,x>0,$ and $a,\omega \in R$ such that $\Re(\delta)>0$ and $\Re(1-\delta-\upsilon)<1+min\{\Re(-\eta),\Re(\alpha+\alpha'-\delta),\Re(\alpha+\eta'-\delta)\}$. Then there holds the following formula:

$$\{I_{-}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{-\upsilon-\delta}S_n^m\left[\omega t^\beta\right]E_{\mu,\upsilon}^{\lambda}\left(at^{-1}\right)\right)\}(x)=\frac{x^{-\upsilon-\alpha-\alpha'}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^\beta\right)^s$$

$$\times {}_4\psi_4\left[\begin{matrix}(\lambda,1),(\upsilon-\beta s+\alpha+\alpha',1),(\upsilon-\beta s+\alpha+\eta',1),(\upsilon-\beta s+\delta-\eta,1)\\(\upsilon,\mu),(\upsilon-\beta s+\delta,1),(\upsilon-\beta s+\alpha+\alpha'+\eta',1),(\upsilon-\beta s+\delta+\alpha-\eta,1)\end{matrix};ax^{-1}\right] \quad (3.16)$$

3.2 Left-sided and right-sided generalized fractional differentiation of product of extended Mittag-Leffler type function and Srivastava polynomials

Now, we shall establish two image formulas for left-sided and right-sided operators of generalized fractional differentiation involving the product of the general class of polinomials (also known as Srivastava polynomials) and extended Mittag-Leffler type function in terms of the generalized Wright hypergeometric function. These formulas are given by the following theorems:

Theorem 3 Let $\alpha,\alpha',\eta,\eta',\delta,\mu,\upsilon,\lambda,p\in C,\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>1,p\geq 0,t,x>0,$ and $a,\omega\in R$ such that $\Re(\delta)>0$ and $\Re(\upsilon)>max\{0,\Re(\delta-\alpha-\alpha'-\eta'),\Re(\eta-\alpha)\}$. Then the left-sided generalized fractional differentiation $D_{0+}^{\alpha,\alpha',\eta,\eta',\delta}$ of the product of the general class of polynomials $S_n^m[z]$ and extended Mittag-Leffler type function $\mathrm{E}_{\mu,\upsilon}^{(\{k_\ell\}_{\ell\in N_0};\lambda)}(z;p)$ is given by

$$\{D_{0+}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\upsilon-1}S_n^m\left[\omega t^\beta\right]\mathrm{E}_{\mu,\upsilon}^{(\{k_\ell\}_{\ell\in N_0};\lambda)}(at;p)\right)\}(x)=\frac{x^{\upsilon-\delta+\alpha+\alpha'-1}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^\beta\right)^s$$

$$\times {}_4\psi_4^{(\{k_\ell\}_{\ell\in N_0})}\left[\begin{matrix}(\lambda,1),(\upsilon+\beta s,1),(\upsilon+\beta s-\delta+\alpha+\alpha'+\eta',1),(\upsilon+\beta s+\alpha-\eta,1)\\(\upsilon,\mu),(\upsilon+\beta s-\delta+\alpha+\alpha',1),(\upsilon+\beta s-\delta+\alpha+\eta',1),(\upsilon+\beta s-\eta,1)\end{matrix};(ax;p)\right] \quad (3.17)$$

Proof: By using (1.4) and (1.12), writing the function in the series form and with the help of (2.3) and (2.15), LHS of Theorem 3 leads to

$$\{D_{0+}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\upsilon-1}S_n^m\left[\omega t^\beta\right]\mathrm{E}_{\mu,\upsilon}^{(\{k_\ell\}_{\ell\in N_0};\lambda)}(at;p)\right)\}(x)$$

$$=\left\{D_{0+}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\upsilon-1}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega t^\beta\right)^s\times\sum_{r=0}^{\infty}\frac{\mathrm{B}^{(\{k_\ell\}_{\ell\in N_0})}(\lambda+r,1-\lambda;p)}{\mathrm{B}(\lambda,1-\lambda)}\times\frac{(at)^r}{\Gamma(\mu r+\upsilon)}\right)\right\}(x),$$

by interchanging the order of differentiation and summations which is valid under the conditions of Theorem 3, we have

$$=\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\,\omega^s\times\sum_{r=0}^{\infty}\frac{\mathrm{B}^{(\{k_\ell\}_{\ell\in N_0})}(\lambda+r,1-\lambda;p)}{\mathrm{B}(\lambda,1-\lambda)}\times\frac{a^r}{\Gamma(\mu r+\upsilon)}\left(D_{0+}^{\alpha,\alpha',\eta,\eta',\delta}t^{\upsilon+\beta s+r-1}\right)(x)$$

$$=\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\,\omega^s\times\sum_{r=0}^{\infty}\frac{\mathrm{B}^{(\{k_\ell\}_{\ell\in N_0})}(\lambda+r,1-\lambda;p)}{\mathrm{B}(\lambda,1-\lambda)}\times\frac{a^r}{\Gamma(\mu r+\upsilon)}\left(I_{0+}^{-\alpha',-\alpha,-\eta',-\eta,-\delta}t^{\upsilon+\beta s+r-1}\right)(x)$$

$$= \frac{x^{\upsilon-\delta+\alpha+\alpha'-1}}{\Gamma(\lambda)} \sum_{s=0}^{[n/m]} \frac{(-n)_{ms}}{s!} A_{n,s} \left(\omega x^{\beta}\right)^{s} \times \sum_{r=0}^{\infty} \frac{\mathrm{B}^{\left(\{k_\ell\}_{\ell\in N_0}\right)}(\lambda+r,1-\lambda;p)}{\mathrm{B}(\lambda,1-\lambda)}$$

$$\times \frac{\Gamma(\upsilon+\beta s+r)\Gamma(\upsilon+\beta s-\delta+\alpha+\alpha'+\eta'+r)\Gamma(\upsilon+\beta s+\alpha-\eta+r)}{\Gamma(\upsilon+\beta s-\delta+\alpha+\alpha'+r)\Gamma(\upsilon+\beta s-\delta+\alpha+\eta'+r)\Gamma(\upsilon+\beta s-\eta+r)} \times \frac{(ax)^{r}}{\Gamma(\mu r+\upsilon)}, \tag{3.18}$$

now, by interpreting the terms of the above equation (3.18), in view of definition(1.9), we arrive at the result (3.17), which completes the proof of theorem 3.

On setting $n=0, A_{0,0}=1$ then $S_0^m[z]\to 1$ in (3.17), we obtain the following result as:

Corollary 3.1 Let $\alpha,\alpha',\eta,\eta',\delta,\mu,\upsilon,\lambda,p\in C, \Re(\mu)>0, \Re(\upsilon)>0, \Re(\lambda)>1, p\geq 0, t,x>0$, and $a,\omega\in R$ such that $\Re(\delta)>0$ and $\Re(\upsilon)>max\{0,\Re(\delta-\alpha-\alpha'-\eta'),\Re(\eta-\alpha)\}$. Then there holds the formula :

$$\{D_{0+}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\upsilon-1}\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(at;p)\right)\}(x)$$

$$= \frac{x^{\upsilon-\delta+\alpha+\alpha'-1}}{\Gamma(\lambda)} \times {}_4\psi_4^{\left(\{k_\ell\}_{\ell\in N_0}\right)}\left[\begin{matrix}(\lambda,1),(\upsilon,1),(\upsilon-\delta+\alpha+\alpha'+\eta',1),(\upsilon+\alpha-\eta,1)\\(\upsilon,\mu),(\upsilon-\delta+\alpha+\alpha',1),(\upsilon-\delta+\alpha+\eta',1),(\upsilon-\eta,1)\end{matrix};(ax;p)\right] \tag{3.19}$$

If we put $\alpha=\alpha+\eta, \alpha'=\eta'=0, \eta=-\delta$ and $\delta=\alpha$, Theorem 3 reduces to the following result:

Corollary 3.2 Let $\alpha,\eta,\delta,\mu,\upsilon,\lambda,p\in C, \Re(\mu)>0, \Re(\upsilon)>0, \Re(\lambda)>1, p\geq 0, t,x>0$ and $a,\omega\in R$ such that $\Re(\alpha)>0$ and $\Re(\upsilon)>max\{0,\Re(\eta-\delta)\}$. Then the following integral formula holds:

$$\{D_{0+}^{\alpha,\eta,\delta}\left(t^{\upsilon-1}S_n^m\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(at;p)\right)\}(x)$$

$$= \frac{x^{\upsilon+\eta-1}}{\Gamma(\lambda)} \sum_{s=0}^{[n/m]} \frac{(-n)_{ms}}{s!} A_{n,s}\left(\omega x^{\beta}\right)^{s} \times {}_3\psi_3^{\left(\{k_\ell\}_{\ell\in N_0}\right)}\left[\begin{matrix}(\lambda,1),(\upsilon+\beta s,1),(\upsilon+\beta s+\delta+\alpha+\eta,1)\\(\upsilon,\mu),(\upsilon+\beta s+\eta,1),(\upsilon+\beta s+\delta,1)\end{matrix};(ax;p)\right] \tag{3.20}$$

Now, if we set $\eta=-\alpha$ in corollary (3.2), we arrive at the following result:

Corollary 3.3 Let $\alpha,\mu,\upsilon,\lambda,p\in C, \Re(\mu)>0, \Re(\upsilon)>0, \Re(\lambda)>1, p\geq 0, t,x>0$ and $a,\omega\in R$ such that $\Re(\alpha)>0$. Then the following result holds:

$$\{D_{0+}^{\alpha}\left(t^{\upsilon-1}S_n^m\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(at;p)\right)\}(x)$$

$$=\frac{x^{\upsilon-\alpha-1}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^{s}\times {}_{2}\psi_{2}^{\left(\{k_{\ell}\}_{\ell\in N_0}\right)}\left[\begin{matrix}(\lambda,1),(\upsilon+\beta s,1)\\(\upsilon,\mu),(\upsilon+\beta s-\alpha,1)\end{matrix};(ax;p)\right] \tag{3.21}$$

Next, if we put $k_{\ell}=\frac{(\rho)_{\ell}}{(\sigma)_{\ell}}(\ell\in N_0)$ in (3.17), then $\mathrm{E}_{\mu,\upsilon}^{\left(\{k_{\ell}\}_{\ell\in N_0};\lambda\right)}(z;p)$ reduce to $\mathrm{E}_{\mu,\upsilon}^{(\rho,\sigma);\lambda}(z;p)$, we arrive at the following result as:

Corollary3.4 Let $\alpha,\alpha',\eta,\eta',\delta,\mu,\upsilon,\lambda,\rho,\sigma,p\in C,\Re(\rho)>0,\Re(\sigma)>0,\Re(\mu)>0,\Re(\upsilon)>0,$ $\Re(\lambda)>1,p\geq 0,t,x>0,$ and $a,\,\omega\in R$ such that $\Re(\delta)>0$ and $\Re(\upsilon)>max\{0,\Re(\delta-\alpha-\alpha'-\eta'),\Re(\eta-\alpha)\}$. Then the following formula holds:

$$\{D_{0+}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\upsilon-1}S_{n}^{m}\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{(\rho,\sigma);\lambda}(at;p)\right)\}(x)=\frac{x^{\upsilon-\delta+\alpha+\alpha'-1}\Gamma(\sigma)}{\Gamma(\lambda)\Gamma(\rho)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^{s}$$

$$\times{}_{5}\psi_{5}\left[\begin{matrix}(\lambda,1),(\upsilon+\beta s,1),(\upsilon+\beta s-\delta+\alpha+\alpha'+\eta',1),(\upsilon+\beta s+\alpha-\eta,1),(\rho,1)\\(\upsilon,\mu),(\upsilon+\beta s-\delta+\alpha+\alpha',1),(\upsilon+\beta s-\delta+\alpha+\eta',1),(\upsilon+\beta s-\eta,1),(\sigma,1)\end{matrix};(ax;p)\right] \tag{3.22}$$

Again, if we set $k_{\ell}=1(\ell\in N_0)$ in (3.17),then $\mathrm{E}_{\mu,\upsilon}^{\left(\{k_{\ell}\}_{\ell\in N_0};\lambda\right)}(z;p)$ reduce to $\mathrm{E}_{\mu,\upsilon}^{\lambda}(z;p)$, we get the following corollary as:

Corollary 3.5 Let $\alpha,\alpha',\eta,\eta',\delta,\mu,\upsilon,\lambda,p\in C,\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>1,p\geq 0,t,x>0,$ and $a,\,\omega\in R$ such that $\Re(\delta)>0$ and $\Re(\upsilon)>max\{0,\Re(\delta-\alpha-\alpha'-\eta'),\Re(\eta-\alpha)\}$. Then the following result holds:

$$\{D_{0+}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\upsilon-1}S_{n}^{m}\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{\lambda}(at;p)\right)\}(x)=\frac{x^{\upsilon-\delta+\alpha+\alpha'-1}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^{s}$$

$$\times{}_{4}\psi_{4}\left[\begin{matrix}(\lambda,1),(\upsilon+\beta s,1),(\upsilon+\beta s-\delta+\alpha+\alpha'+\eta',1),(\upsilon+\beta s+\alpha-\eta,1)\\(\upsilon,\mu),(\upsilon+\beta s-\delta+\alpha+\alpha',1),(\upsilon+\beta s-\delta+\alpha+\eta',1),(\upsilon+\beta s-\eta,1)\end{matrix};(ax;p)\right] \tag{3.23}$$

Further, if we take $k_{\ell}=1(\ell\in N_0)\,and\;p=0$ in (3.1), then $\mathrm{E}_{\mu,\upsilon}^{\left(\{k_{\ell}\}_{\ell\in N_0};\lambda\right)}(z;p)$ reduce to $\mathrm{E}_{\mu,\upsilon}^{\lambda}(z)$, then we obtain the following result as:

Corollary 3.6 Let $\alpha,\alpha',\eta,\eta',\delta,\mu,\upsilon,\lambda\in C,\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>1,t,x>0,$ and $a,\,\omega\in R$ such that $\Re(\delta)>0$ and $\Re(\upsilon)>max\{0,\Re(\delta-\alpha-\alpha'-\eta'),\Re(\eta-\alpha)\}$. Then there holds the following formula true:

$$\{D_{0+}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\upsilon-1}S_n^m\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{\lambda}\left(at\right)\right)\}\left(x\right)=\frac{x^{\upsilon-\delta+\alpha+\alpha'-1}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{\left(-n\right)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^{s}$$

$$\times{}_4\psi_4\left[\begin{matrix}(\lambda,1),(\upsilon+\beta s,1),(\upsilon+\beta s-\delta+\alpha+\alpha'+\eta',1),(\upsilon+\beta s+\alpha-\eta,1)\\(\upsilon,\mu),(\upsilon+\beta s-\delta+\alpha+\alpha',1),(\upsilon+\beta s-\delta+\alpha+\eta',1),(\upsilon+\beta s-\eta,1)\end{matrix};ax\right] \tag{3.24}$$

Theorem 4 Let $\alpha,\alpha',\eta,\eta',\delta,\mu,\upsilon,\lambda,p\in C,\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>1,p\geq 0,t,x>0$, and $a,\ \omega\in R$ such that $\Re(\delta)>0$ and $\Re(1-\delta-\upsilon)<1+min\{\Re(\eta'),\Re(\delta-\alpha-\alpha'),\Re(\delta-\alpha'-\eta)\}$. Then the right-sided generalized fractional differentiation $D_{-}^{\alpha,\alpha',\eta,\eta',\delta}$ of the product of the general class of polynomials $S_n^m[z]$ and extended Mittag-Leffler type function $\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(z;p)$ is given by

$$\{D_{-}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\delta-\upsilon}S_n^m\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}\left(at^{-1};p\right)\right)\}\left(x\right)=\frac{x^{-\upsilon+\alpha+\alpha'}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{\left(-n\right)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^{s}$$

$$\times{}_4\psi_4^{\left(\{k_\ell\}_{\ell\in N_0}\right)}\left[\begin{matrix}(\lambda,1),(\upsilon-\beta s-\alpha-\alpha',1),(\upsilon-\beta s-\alpha'-\eta,1),(\upsilon-\beta s-\delta+\eta',1)\\(\upsilon,\mu),(\upsilon-\beta s-\delta,1),(\upsilon-\beta s-\alpha-\alpha'-\eta,1),(\upsilon-\beta s-\delta-\alpha'+\eta',1)\end{matrix};\left(ax^{-1};p\right)\right] \tag{3.25}$$

Proof: By using (1.4) and (1.12) writing the function in the series form and with the help of (2.5) and (2.16), the LHS of (3.25) can be written as

$$\{D_{-}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\delta-\upsilon}S_n^m\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}\left(at^{-1};p\right)\right)\}\left(x\right)$$

$$=\left\{D_{-}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\delta-\upsilon}\sum_{s=0}^{[n/m]}\frac{\left(-n\right)_{ms}}{s!}A_{n,s}\left(\omega t^{\beta}\right)^{s}\times\sum_{r=0}^{\infty}\frac{\mathrm{B}^{\left(\{k_\ell\}_{\ell\in N_0}\right)}\left(\lambda+r,1-\lambda;p\right)}{\mathrm{B}\left(\lambda,1-\lambda\right)}\times\frac{\left(at^{-1}\right)^{r}}{\Gamma\left(\mu r+\upsilon\right)}\right)\right\}\left(x\right),$$

by interchanging the order of differentiation and summations, which is valid under the conditions of Theorem 4, we have

$$=\sum_{s=0}^{[n/m]}\frac{\left(-n\right)_{ms}}{s!}A_{n,s}\ \omega^{s}\times\sum_{r=0}^{\infty}\frac{\mathrm{B}^{\left(\{k_\ell\}_{\ell\in N_0}\right)}\left(\lambda+r,1-\lambda;p\right)}{\mathrm{B}\left(\lambda,1-\lambda\right)}\times\frac{a^{r}}{\Gamma\left(\mu r+\upsilon\right)}\left(D_{-}^{\alpha,\alpha',\eta,\eta',\delta}t^{\delta-\upsilon+\beta s-r}\right)\left(x\right)$$

$$=\sum_{s=0}^{[n/m]}\frac{\left(-n\right)_{ms}}{s!}A_{n,s}\ \omega^{s}\times\sum_{r=0}^{\infty}\frac{\mathrm{B}^{\left(\{k_\ell\}_{\ell\in N_0}\right)}\left(\lambda+r,1-\lambda;p\right)}{\mathrm{B}\left(\lambda,1-\lambda\right)}\times\frac{a^{r}}{\Gamma\left(\mu r+\upsilon\right)}\left(I_{-}^{-\alpha',-\alpha,-\eta',-\eta,-\delta}t^{(1+\delta-\upsilon+\beta s-r)-1}\right)\left(x\right)$$

$$=\frac{x^{-\upsilon+\alpha+\alpha'}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^{s}\times\sum_{r=0}^{\infty}\frac{\mathrm{B}^{\left(\{k_\ell\}_{\ell\in N_0}\right)}\left(\lambda+r,1-\lambda;p\right)}{\mathrm{B}\left(\lambda,1-\lambda\right)}$$

$$\times\frac{\Gamma\left(\upsilon-\beta s-\alpha-\alpha'+r\right)\Gamma\left(\upsilon-\beta s-\alpha'-\eta+r\right)\Gamma\left(\upsilon-\beta s-\delta+\eta'+r\right)}{\Gamma\left(\upsilon-\beta s-\delta+r\right)\Gamma\left(\upsilon-\beta s-\alpha-\alpha'-\eta+r\right)\Gamma\left(\upsilon-\beta s-\delta-\alpha'+\eta'+r\right)}\times\frac{\left(ax^{-1}\right)^{r}}{\Gamma\left(\mu r+\upsilon\right)},\quad(3.26)$$

now, by interpreting the terms of the above equation (3.26), in view of definition(1.9), we arrive at the result (3.25), which completes the proof of theorem 4.

On setting $n=0, A_{0,0}=1$ then $S_0^m[z]\to 1$ in equation (3.25), we get the following result as:

Corollary 4.1 Let $\alpha,\alpha',\eta,\eta',\delta,\mu,\upsilon,\lambda,p\in C,\Re\left(\mu\right)>0,\Re\left(\upsilon\right)>0,\Re\left(\lambda\right)>1,p\geq 0,t,x>0,$ and $a,\omega\in R$ such that $\Re\left(\delta\right)>0$ and $\Re\left(1-\delta-\upsilon\right)<1+min\{\Re\left(\eta'\right),\Re\left(\delta-\alpha-\alpha'\right),\Re\left(\delta-\alpha'-\eta\right)\}$. Then there holds the followig formula:

$$\{D_{-}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\delta-\upsilon}\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}\left(at^{-1};p\right)\right)\}\left(x\right)$$

$$=\frac{x^{-\upsilon+\alpha+\alpha'}}{\Gamma(\lambda)}\times{}_4\psi_4^{\left(\{k_\ell\}_{\ell\in N_0}\right)}\left[\begin{matrix}(\lambda,1),(\upsilon-\alpha-\alpha',1),(\upsilon-\alpha'-\eta,1),(\upsilon-\delta+\eta',1)\\(\upsilon,\mu),(\upsilon-\delta,1),(\upsilon-\alpha-\alpha'-\eta,1),(\upsilon-\delta-\alpha'+\eta',1)\end{matrix};\left(ax^{-1};p\right)\right]\quad(3.27)$$

If we take $\alpha=\alpha+\eta,\alpha'=\eta'=0,\eta=-\delta$ and $\delta=\alpha$, Theorem 4 reduces to the following corollary:

Corollary 4.2 Let $\alpha,\eta,\delta,\mu,\upsilon,\lambda,p\in C,\Re\left(\mu\right)>0,\Re\left(\upsilon\right)>0,\Re\left(\lambda\right)>1,p\geq 0,t,x>0$ and $a,\omega\in R$ such that $\Re\left(\alpha\right)>0$ and $\Re\left(1-\delta-\upsilon\right)<1+min\left\{\Re\left(-\eta\right),\Re\left(-\delta\right)\right\}$. Then we have

$$\{D_{-}^{\alpha,\eta,\delta}\left(t^{\alpha-\upsilon}S_n^m\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}\left(at^{-1};p\right)\right)\}\left(x\right)=\frac{x^{-\upsilon+\alpha+\eta}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^{s}$$

$$\times{}_3\psi_3^{\left(\{k_\ell\}_{\ell\in N_0}\right)}\left[\begin{matrix}(\lambda,1),(\upsilon-\beta s-\alpha-\eta,1),(\upsilon-\beta s+\delta,1)\\(\upsilon,\mu),(\upsilon-\beta s-\alpha,1),(\upsilon-\beta s+\delta-\alpha-\eta,1)\end{matrix};\left(ax^{-1};p\right)\right]\quad(3.28)$$

Now, if we take $\eta=-\alpha$ in corollary (4.2), we arrive at the following result:

Corollary 4.3 Let $\alpha,\mu,\upsilon,\lambda,p\in C,\Re\left(\mu\right)>0,\Re\left(\upsilon\right)>0,\Re\left(\lambda\right)>1,p\geq 0,t,x>0$ and $a,\ \omega\in R$ such that $\Re\left(\alpha\right)>0$. Then the following result holds:

$$\{D_{-}^{\alpha}\left(t^{\alpha-\upsilon}S_n^m\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}\left(at^{-1};p\right)\right)\}\left(x\right)$$

$$=\frac{x^{-\upsilon}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^{s}\times{}_{2}\psi_{2}^{\left(\{k_\ell\}_{\ell\in N_0}\right)}\left[\begin{matrix}(\lambda,1),(\upsilon-\beta s,1)\\(\upsilon,\mu),(\upsilon-\beta s-\alpha,1)\end{matrix};\left(ax^{-1};p\right)\right] \tag{3.29}$$

Next, if we put $k_\ell=\frac{(\rho)_\ell}{(\sigma)_\ell}(\ell\in N_0)$ in (3.25), then $\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(z;p)$ reduce to $\mathrm{E}_{\mu,\upsilon}^{(\rho,\sigma);\lambda}(z;p)$, we have the following formula as:

Corollary4.4 Let $\alpha,\alpha',\eta,\eta',\delta,\rho,\sigma\mu,\upsilon,\lambda,p\in C,\Re(\rho)>0,\Re(\sigma)>0,\Re(\mu)>0,\Re(\upsilon)>0,$ $\Re(\lambda)>1,p\geq 0,t,x>0,$ and $a,\omega\in R$ such that $\Re(\delta)>0$ and $\Re(1-\delta-\upsilon)<1+min\{\Re(\eta'),\Re(\delta-\alpha-\alpha'),\Re(\delta-\alpha'-\eta)\}$. Then the following result holds:

$$\{D_{-}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\delta-\upsilon}S_n^m\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{(\rho,\sigma);\lambda}\left(at^{-1};p\right)\right)\}(x)=\frac{x^{-\upsilon+\alpha+\alpha'}\Gamma(\sigma)}{\Gamma(\lambda)\Gamma(\rho)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^{s}$$

$$\times{}_{5}\psi_{5}\left[\begin{matrix}(\lambda,1),(\upsilon-\beta s-\alpha-\alpha',1),(\upsilon-\beta s-\alpha'-\eta,1),(\upsilon-\beta s-\delta+\eta',1),(\rho,1)\\(\upsilon,\mu),(\upsilon-\beta s-\delta,1),(\upsilon-\beta s-\alpha-\alpha'-\eta,1),(\upsilon-\beta s-\delta-\alpha'+\eta',1),(\sigma,1)\end{matrix};\left(ax^{-1};p\right)\right]. \tag{3.30}$$

Again, if we set $k_\ell=1(\ell\in N_0)$ in (3.25), then $\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(z;p)$ reduce to $\mathrm{E}_{\mu,\upsilon}^{\lambda}(z;p)$, we obtain the following result as

Corollary 4.5 Let $\alpha,\alpha',\eta,\eta',\delta,\mu,\upsilon,\lambda,p\in C,\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>1,p\geq 0,t,x>0,$ and $a,\omega\in R$ such that $\Re(\delta)>0$ and $\Re(1-\delta-\upsilon)<1+min\{\Re(\eta'),\Re(\delta-\alpha-\alpha'),\Re(\delta-\alpha'-\eta)\}$. Then the following formula holds:

$$\{D_{-}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\delta-\upsilon}S_n^m\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{\lambda}\left(at^{-1};p\right)\right)\}(x)=\frac{x^{-\upsilon+\alpha+\alpha'}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^{s}$$

$$\times{}_{4}\psi_{4}\left[\begin{matrix}(\lambda,1),(\upsilon-\beta s-\alpha-\alpha',1),(\upsilon-\beta s-\alpha'-\eta,1),(\upsilon-\beta s-\delta+\eta',1)\\(\upsilon,\mu),(\upsilon-\beta s-\delta,1),(\upsilon-\beta s-\alpha-\alpha'-\eta,1),(\upsilon-\beta s-\delta-\alpha'+\eta',1)\end{matrix};\left(ax^{-1};p\right)\right]. \tag{3.31}$$

Further, if we take $k_\ell=1(\ell\in N_0)$ *and* $p=0$ in (3.25), then $\mathrm{E}_{\mu,\upsilon}^{\left(\{k_\ell\}_{\ell\in N_0};\lambda\right)}(z;p)$ reduce to $\mathrm{E}_{\mu,\upsilon}^{\lambda}(z)$, then we obtain the following result as:

Corollary 4.6 Let $a,\alpha,\alpha',\eta,\eta',\delta,\mu,\upsilon,\lambda\in C,\Re(\mu)>0,\Re(\upsilon)>0,\Re(\lambda)>1,t,x>0,$ and $a,\omega\in R$ such that $\Re(\delta)>0$ and $\Re(1-\delta-\upsilon)<1+min\{\Re(\eta'),\Re(\delta-\alpha-\alpha'),\Re(\delta-\alpha'-\eta)\}$. Then there holds the following formula:

$$\{D_{-}^{\alpha,\alpha',\eta,\eta',\delta}\left(t^{\delta-\upsilon}S_n^m\left[\omega t^{\beta}\right]\mathrm{E}_{\mu,\upsilon}^{\lambda}\left(at^{-1}\right)\right)\}(x)=\frac{x^{-\upsilon+\alpha+\alpha'}}{\Gamma(\lambda)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(\omega x^{\beta}\right)^{s}$$

$$\times{}_4\psi_4\left[\begin{matrix}(\lambda,1),(\upsilon-\beta s-\alpha-\alpha',1),(\upsilon-\beta s-\alpha'-\eta,1),(\upsilon-\beta s-\delta+\eta',1)\\(\upsilon,\mu),(\upsilon-\beta s-\delta,1),(\upsilon-\beta s-\alpha-\alpha'-\eta,1),(\upsilon-\beta s-\delta-\alpha'+\eta',1)\end{matrix};ax^{-1}\right]. \tag{3.32}$$

4. Conclusion

In this paper, we have established four image formulas for generalized fractional calculus operators involving the product of the general class of polynomials (known as Srivastava polynomials) and the extended Mittag-Leffler type function. The final results are expressed in terms of the generalized Wright hypergeometric function. The main advantage of these formulas lies in their generality. Since the Srivastava polynomials and the extended Mittag-Leffler function include many simpler special functions as particular cases, our main results can be used to derive several new and existing image formulas. By choosing appropriate values for the parameters, one can easily obtain corresponding results for classical functions such as the Jacobi, Laguerre, Hermite, Bessel, and classical Mittag-Leffler functions. In conclusion, the results obtained in this study contribute to the theory of fractional calculus and provide useful tools for researchers working in mathematical analysis, fractional differential equations, and related fields.

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