A theorem on general matrix summability method

ABSTRACT. The paper presents a new general theorem on $\phi - |B;\delta|_k$ summability of an infinite series, where B is a positive normal matrix, (ϕ_n) is any sequence of positive real numbers, $k \geq 1$ and $\delta \geq 0$. The proposed theorem provides a broader framework for the study of generalized matrix summability methods and extends existing results under suitable assumptions.
Keywords:summability factor, absolute matrix summability, infinite series, matrix transformation.
Mathematics Subject Classification (2020): 40D15, 40F05, 40G99.

1 Introduction

Consider an infinite series $\sum a_n$ with the partial sums (s_n) and $B = (b_{nv})$ be a normal matrix, i.e., a lower triangular matrix of non-zero diagonal entries. Then B defines the sequence-to-sequence transformation, mapping the sequence $s = (s_n)$ to $Bs = (B_n(s))$, where $B_n(s) = \sum_{v=0}^{n} b_{nv} s_v, \quad n = 0, 1, ...$ Let (ϕ_n) be any sequence of positive real numbers. The series $\sum a_n$ is said to be $\phi - |B;\delta|_k$ summable, $k \geq 1$ and $\delta \geq 0$, if (see [9])

$$\sum_{n=1}^{\infty} \phi_n^{\delta k+k-1} |B_n(s) - B_{n-1}(s)|^k < \infty.$$

For $\delta = 0$, $\phi_n = \frac{P_n}{p_n}$ and $b_{nv} = \frac{p_v}{P_n}$, $|\bar{N}, p_n|_k$ summability (see [2]) is obtained.

2 Known Results

Concerning $|\bar{N}, p_n|_k$ summability, Bor [1] proved the following theorem.

Theorem 1 *Let the sequences* (λ_n) *and* (p_n) *satisfy the following conditions:*

$$\sum_{n=2}^{m} \frac{p_n|\lambda_n|}{P_n} = O(1), \tag{1}$$

$$P_m|\Delta\lambda_m| = O(p_m|\lambda_m|) \quad as \quad m \to \infty. \tag{2}$$

If

$$\sum_{v=1}^{n} p_v|s_v|^k = O(P_n\gamma_n) \quad as \quad n \to \infty \tag{3}$$

where (γ_n) *is a positive non-decreasing sequence such that*

$$P_{n+1}\gamma_n\Delta\left(\frac{1}{\gamma_n}\right) = O(p_{n+1}) \quad as \quad n \to \infty \tag{4}$$

then the series $\sum a_n\lambda_n(\gamma_n)^{-1}$ *is summable* $|\bar{N}, p_n|_k$, $k \geq 1$

Theorem 2 *[3] Let the sequences* (λ_n) *and* (p_n) *satisfy the condition (2) and*

$$\sum_{n=2}^{m} \frac{p_n|\lambda_n|^k}{P_n} = O(1) \quad as \quad m \to \infty. \tag{5}$$

If the condition (3) satisfies, then the series $\sum a_n\lambda_n(\gamma_n)^{-1}$ *is summable* $|\bar{N}, p_n|_k$, *where* $k \geq 1$ *and* (γ_n) *is as in Theorem 1.*

Lemma 1 *[4] If the sequences* (λ_n) *and* (p_n) *satisfy the conditions (2) and (5), then*

$$|\lambda_n| = O(1), \tag{6}$$

$$P_n\Delta\left(|\lambda_n|^k\right) = O(p_n|\lambda_n|^k) \quad as \quad n \to \infty. \tag{7}$$

3 Main Result

The main contribution of this paper is the establishment of a new theorem for the generalized $\phi - |B;\delta|_k$ summability method associated with positive normal matrices. Compared with the existing results, the proposed theorem extends the study to a broader matrix setting and provides a unified framework for generalized matrix summability methods under suitable assumptions. Several studies have been devoted to generalized absolute matrix summability methods, where various summability factor theorems have been established for positive normal matrices and related matrix transformations; see [5–8]. On the other hand, similar summability factor results have been obtained by employing different classes of sequences, including almost increasing sequences, and δ-quasi-monotone sequences; see [10–16]. Let $B = (b_{nv})$ be a normal matrix, we associate two lower semimatrices $\bar{B} = (\bar{b}_{nv})$ and $\hat{B} = (\hat{b}_{nv})$ as follows:

$$\bar{b}_{nv} = \sum_{i=v}^{n} b_{ni}, \quad n, v = 0, 1, ... \tag{8}$$

$$\hat{b}_{00} = \bar{b}_{00} = b_{00}, \quad \hat{b}_{nv} = \bar{b}_{nv} - \bar{b}_{n-1,v}, \quad n = 1, 2, ... \tag{9}$$

and

$$B_n(s) = \sum_{v=0}^{n} b_{nv} s_v = \sum_{v=0}^{n} \bar{b}_{nv} a_v \tag{10}$$

$$\bar{\Delta} B_n(s) = \sum_{v=0}^{n} \hat{b}_{nv} a_v. \tag{11}$$

It should be noted that

$$a_n \asymp b_n$$

if there exist constants $C_1, C_2 > 0$ such that

$$C_1 b_n \leq a_n \leq C_2 b_n$$

for all sufficiently large n.

Now, let establish the following theorem.

Theorem 3 *Let $p_{v+1}/P_{v+1} = O\left(p_v/P_v\right)$ and $\phi_n p_n \asymp P_n$. Let all conditions of Theorem 2 be satisfied with the condition (3) replaced by*

$$\sum_{v=1}^{n} \phi_v^{\delta k} p_v |s_v|^k = O(P_n \gamma_n) \quad as \quad n \to \infty. \tag{12}$$

If $B = (b_{nv})$ is a positive normal matrix such that

$$\bar{b}_{n0} = 1, \; n = 0, 1, ..., \tag{13}$$

$$b_{n-1,v} \geq b_{nv}, \; for \; n \geq v + 1, \tag{14}$$

$$b_{nn} = O\left(\frac{p_n}{P_n}\right), \tag{15}$$

$$\sum_{n=2}^{m} \frac{p_n}{P_n} = O(1) \quad as \quad m \to \infty. \tag{16}$$

$$\sum_{n=v+1}^{m+1} \phi_n^{\delta k} |\hat{b}_{n,v+1}| = O\left(\phi_v^{\delta k}\right) \quad as \quad m \to \infty, \tag{17}$$

$$\sum_{n=v+1}^{m+1} \phi_n^{\delta k} |\Delta_v \left(\hat{b}_{nv}\right)| = O\left(\phi_v^{\delta k-1}\right) \quad as \quad m \to \infty, \tag{18}$$

where $\Delta_v(\hat{b}_{nv}) = \hat{b}_{nv} - \hat{b}_{n,v+1}$*, then the series* $\sum a_n \lambda_n (\gamma_n)^{-1}$ *is summable* $\phi - |B;\delta|_k$*,* $k \geq 1$ *and* $0 \leq \delta < 1/k$*.*

4 Proof of Theorem 3

Let (D_n) be the sequence of B-transform of the series $\sum a_n \lambda_n (\gamma_n)^{-1}$. Then, by (11), we have

$$\bar{\Delta} D_n = \sum_{v=1}^{n} \hat{b}_{nv} a_v \lambda_v (\gamma_v)^{-1}.$$

By using Abel's transformation for the above sum, we obtain

$$\begin{aligned}
\bar{\Delta} D_n &= \sum_{v=1}^{n-1} \Delta_v(\hat{b}_{nv} \lambda_v (\gamma_v)^{-1}) \sum_{r=1}^{v} a_r + \hat{b}_{nn} \lambda_n (\gamma_n)^{-1} \sum_{v=1}^{n} a_v \\
&= \sum_{v=1}^{n-1} \Delta_v(\hat{b}_{nv} \lambda_v (\gamma_v)^{-1}) s_v + \hat{b}_{nn} \lambda_n (\gamma_n)^{-1} s_n \\
&= b_{nn} \lambda_n (\gamma_n)^{-1} s_n + \sum_{v=1}^{n-1} \Delta_v(\hat{b}_{nv}) \lambda_v (\gamma_v)^{-1} s_v + \sum_{v=1}^{n-1} \hat{b}_{n,v+1} \Delta \lambda_v (\gamma_{v+1})^{-1} s_v \\
&\quad + \sum_{v=1}^{n-1} \hat{b}_{n,v+1} \lambda_v \Delta \left((\gamma_v)^{-1}\right) s_v \\
&= D_{n,1} + D_{n,2} + D_{n,3} + D_{n,4}.
\end{aligned}$$

First, by (12) and (15), we achieve

$$\begin{aligned}
\sum_{n=1}^{m}\phi_n^{\delta k+k-1}|D_{n,1}|^k &= \sum_{n=1}^{m}\phi_n^{\delta k+k-1}b_{nn}^k|\lambda_n|^k\gamma_n^{-k}|s_n|^k \\
&= O(1)\sum_{n=1}^{m}\phi_n^{\delta k}\frac{|\lambda_n|^k p_n|s_n|^k}{P_n\gamma_n} \\
&= O(1)\sum_{n=1}^{m-1}\Delta\left(\frac{|\lambda_n|^k}{P_n\gamma_n}\right)\sum_{r=1}^{n}\phi_r^{\delta k}p_r\,|s_r|^k + O(1)\frac{|\lambda_m|^k}{P_m\gamma_m}\sum_{n=1}^{m}\phi_n^{\delta k}p_n\,|s_n|^k \\
&= O(1)\sum_{n=1}^{m-1}\Delta\left(\frac{|\lambda_n|^k}{P_n\gamma_n}\right)P_n\gamma_n + O(1)\frac{|\lambda_m|^k}{P_m\gamma_m}P_m\gamma_m \\
&= O(1)\sum_{n=1}^{m-1}\Delta\left(|\lambda_n|^k\right) + O(1)\sum_{n=1}^{m-1}|\lambda_{n+1}|^k\,\gamma_n\Delta\left(\frac{1}{\gamma_n}\right) + O(1)\sum_{n=1}^{m-1}\frac{p_{n+1}|\lambda_{n+1}|^k}{P_{n+1}} \\
&+ O(1)|\lambda_m|^k.
\end{aligned}$$

Then by (4), (5), (6), (7), we get

$$\begin{aligned}
\sum_{n=1}^{m}\phi_n^{\delta k+k-1}|D_{n,1}|^k &= O(1)\sum_{n=1}^{m-1}\frac{p_n|\lambda_n|^k}{P_n} + O(1)\sum_{n=1}^{m-1}\frac{p_{n+1}|\lambda_{n+1}|^k}{P_{n+1}} + O(1)\sum_{n=1}^{m-1}\frac{p_{n+1}|\lambda_{n+1}|^k}{P_{n+1}} + O(1) \\
&= O(1) \quad as \quad m\to\infty.
\end{aligned}$$

Next, by Hölder's inequality, (15), (18), we obtain

$$\begin{aligned}
\sum_{n=2}^{m+1} \phi_n^{\delta k+k-1} |D_{n,2}|^k &\leq \sum_{n=2}^{m+1} \phi_n^{\delta k+k-1} \left(\sum_{v=1}^{n-1} \left| \Delta_v(\hat{b}_{nv}) \right| \frac{|\lambda_v|\,|s_v|}{\gamma_v} \right)^k \\
&\leq \sum_{n=2}^{m+1} \phi_n^{\delta k+k-1} \left(\sum_{v=1}^{n-1} \left| \Delta_v(\hat{b}_{nv}) \right| \frac{|\lambda_v|^k\,|s_v|^k}{\gamma_v^k} \right) \left(\sum_{v=1}^{n-1} |\Delta_v(\hat{b}_{nv})| \right)^{k-1} \\
&\leq \sum_{n=2}^{m+1} \phi_n^{\delta k+k-1} b_{nn}^{k-1} \left(\sum_{v=1}^{n-1} \left| \Delta_v(\hat{b}_{nv}) \right| \frac{|\lambda_v|^k\,|s_v|^k}{\gamma_v^k} \right) \\
&= O(1) \sum_{v=1}^{m} \frac{|\lambda_v|^k\,|s_v|^k}{\gamma_v^k} \sum_{n=v+1}^{m+1} \phi_n^{\delta k} |\Delta_v(\hat{b}_{nv})| \\
&= O(1) \sum_{v=1}^{m} \phi_v^{\delta k} \frac{|\lambda_v|^k\,p_v\,|s_v|^k}{P_v \gamma_v} \\
&= O(1) \sum_{v=1}^{m-1} \Delta \left(\frac{|\lambda_v|^k}{P_v \gamma_v} \right) \sum_{r=1}^{v} \phi_r^{\delta k} p_r\,|s_r|^k + O(1) \frac{|\lambda_m|^k}{P_m \gamma_m} \sum_{v=1}^{m} \phi_v^{\delta k} p_v\,|s_v|^k \\
&= O(1) \sum_{v=1}^{m-1} \Delta \left(\frac{|\lambda_v|^k}{P_v \gamma_v} \right) P_v \gamma_v + O(1) \frac{|\lambda_m|^k}{P_m \gamma_m} P_m \gamma_m \\
&= O(1) \sum_{v=1}^{m-1} \Delta \left(|\lambda_v|^k \right) + O(1) \sum_{v=1}^{m-1} |\lambda_{v+1}|^k \gamma_v \Delta \left(\frac{1}{\gamma_v} \right) + O(1) \sum_{v=1}^{m-1} \frac{p_{v+1} |\lambda_{v+1}|^k}{P_{v+1}} \\
&+ O(1) |\lambda_m|^k \\
&= O(1) \sum_{v=1}^{m-1} \frac{p_v |\lambda_v|^k}{P_v} + O(1) \sum_{v=1}^{m-1} \frac{p_{v+1} |\lambda_{v+1}|^k}{P_{v+1}} + O(1) \sum_{v=1}^{m-1} \frac{p_{v+1} |\lambda_{v+1}|^k}{P_{v+1}} + O(1) \\
&= O(1) \quad as \quad m \to \infty.
\end{aligned}$$

Now, by (2), we get

$$\begin{aligned}
\sum_{n=2}^{m+1} \phi_n^{\delta k+k-1} |D_{n,3}|^k &\leq \sum_{n=2}^{m+1} \phi_n^{\delta k+k-1} \left(\sum_{v=1}^{n-1} |\hat{b}_{n,v+1}| \frac{|\Delta \lambda_v||s_v|}{\gamma_v} \right)^k \\
&\leq \sum_{n=2}^{m+1} \phi_n^{\delta k+k-1} \left(\sum_{v=1}^{n-1} |\hat{b}_{n,v+1}|^k |s_v|^k\,|\lambda_v|^k\,\frac{1}{\gamma_v^k} \frac{p_v}{P_v} \right) \left(\sum_{v=1}^{n-1} \frac{p_v}{P_v} \right)^{k-1}.
\end{aligned}$$

Here, the condition (16) gives

$$\sum_{n=2}^{m+1} \phi_n^{\delta k+k-1} |D_{n,3}|^k = O(1) \sum_{n=2}^{m+1} \phi_n^{\delta k+k-1} \left(\sum_{v=1}^{n-1} |\hat{b}_{n,v+1}|^{k-1} |\hat{b}_{n,v+1}| \frac{|\lambda_v|^k\,p_v\,|s_v|^k}{P_v \gamma_v} \right).$$

Thence, by (15) and (17), we get

$$\begin{aligned}
\sum_{n=2}^{m+1} \phi_n^{\delta k+k-1} |D_{n,3}|^k &= O(1) \sum_{n=2}^{m+1} \phi_n^{\delta k+k-1} b_{nn}^{k-1} \left(\sum_{v=1}^{n-1} |\hat{b}_{n,v+1}| \frac{|\lambda_v|^k \, p_v \, |s_v|^k}{P_v \gamma_v} \right) \\
&= O(1) \sum_{v=1}^{m} \frac{|\lambda_v|^k \, p_v \, |s_v|^k}{P_v \gamma_v} \sum_{n=v+1}^{m+1} \phi_n^{\delta k} |\hat{b}_{n,v+1}| \\
&= O(1) \sum_{v=1}^{m} \phi_v^{\delta k} \frac{|\lambda_v|^k \, p_v \, |s_v|^k}{P_v \gamma_v} \\
&= O(1) \sum_{v=1}^{m-1} \Delta \left(\frac{|\lambda_v|^k}{P_v \gamma_v} \right) \sum_{r=1}^{v} \phi_r^{\delta k} p_r \, |s_r|^k + O(1) \frac{|\lambda_m|^k}{P_m \gamma_m} \sum_{v=1}^{m} \phi_v^{\delta k} p_v \, |s_v|^k \\
&= O(1) \sum_{v=1}^{m-1} \Delta \left(\frac{|\lambda_v|^k}{P_v \gamma_v} \right) P_v \gamma_v + O(1) \frac{|\lambda_m|^k}{P_m \gamma_m} P_m \gamma_m \\
&= O(1) \sum_{v=1}^{m-1} \Delta \left(|\lambda_v|^k \right) + O(1) \sum_{v=1}^{m-1} |\lambda_{v+1}|^k \gamma_v \Delta \left(\frac{1}{\gamma_v} \right) + O(1) \sum_{v=1}^{m-1} \frac{p_{v+1} |\lambda_{v+1}|^k}{P_{v+1}} \\
&+ O(1) |\lambda_m|^k \\
&= O(1) \sum_{v=1}^{m-1} \frac{p_v |\lambda_v|^k}{P_v} + O(1) \sum_{v=1}^{m-1} \frac{p_{v+1} |\lambda_{v+1}|^k}{P_{v+1}} + O(1) \sum_{v=1}^{m-1} \frac{p_{v+1} |\lambda_{v+1}|^k}{P_{v+1}} + O(1) \\
&= O(1) \quad as \quad m \to \infty.
\end{aligned}$$

Finally, we obtain

$$\begin{aligned}
\sum_{n=2}^{m+1} \phi_n^{\delta k+k-1} |D_{n,4}|^k &\le \sum_{n=2}^{m+1} \phi_n^{\delta k+k-1} \left(\sum_{v=1}^{n-1} |\hat{b}_{n,v+1}| |\lambda_v| |s_v| \Delta \left(\frac{1}{\gamma_v} \right) \right)^k \\
&= O(1) \sum_{n=2}^{m+1} \phi_n^{\delta k+k-1} \left(\sum_{v=1}^{n-1} |\hat{b}_{n,v+1}| |\lambda_v| |s_v| \frac{p_{v+1}}{P_{v+1} \gamma_v} \right)^k \\
&= O(1) \sum_{n=2}^{m+1} \phi_n^{\delta k+k-1} \left(\sum_{v=1}^{n-1} |\hat{b}_{n,v+1}|^k \, |\lambda_v|^k \, |s_v|^k \, \frac{1}{\gamma_v^k} \frac{p_{v+1}}{P_{v+1}} \right) \left(\sum_{v=1}^{n-1} \frac{p_{v+1}}{P_{v+1}} \right)^{k-1}.
\end{aligned}$$

Now, using (16), we get

$$\sum_{n=2}^{m+1} \phi_n^{\delta k+k-1} |D_{n,4}|^k = O(1) \sum_{n=2}^{m+1} \phi_n^{\delta k+k-1} b_{nn}^{k-1} \left(\sum_{v=1}^{n-1} |\hat{b}_{n,v+1}| \frac{|\lambda_v|^k \, p_{v+1} \, |s_v|^k}{P_{v+1} \gamma_v} \right).$$

Then, we get

$$\begin{aligned}
\sum_{n=2}^{m+1}\phi_n^{\delta k+k-1}|D_{n,4}|^k &= O(1)\sum_{v=1}^{m}\frac{|\lambda_v|^k\,p_{v+1}\,|s_v|^k}{P_{v+1}\gamma_v}\sum_{n=v+1}^{m+1}\phi_n^{\delta k}|\hat{b}_{n,v+1}|\\
&= O(1)\sum_{v=1}^{m}\phi_v^{\delta k}\frac{|\lambda_v|^k\,p_{v+1}\,|s_v|^k}{P_{v+1}\gamma_v}\\
&= O(1)\sum_{v=1}^{m}\phi_v^{\delta k}\frac{|\lambda_v|^k\,p_v\,|s_v|^k}{P_v\gamma_v}\\
&= O(1)\sum_{v=1}^{m-1}\Delta\left(\frac{|\lambda_v|^k}{P_v\gamma_v}\right)\sum_{r=1}^{v}\phi_r^{\delta k}p_r\,|s_r|^k+O(1)\frac{|\lambda_m|^k}{P_m\gamma_m}\sum_{v=1}^{m}\phi_v^{\delta k}p_v\,|s_v|^k\\
&= O(1)\sum_{v=1}^{m-1}\Delta\left(\frac{|\lambda_v|^k}{P_v\gamma_v}\right)P_v\gamma_v+O(1)\frac{|\lambda_m|^k}{P_m\gamma_m}P_m\gamma_m\\
&= O(1)\sum_{v=1}^{m-1}\Delta\left(|\lambda_v|^k\right)+O(1)\sum_{v=1}^{m-1}|\lambda_{v+1}|^k\,\gamma_v\Delta\left(\frac{1}{\gamma_v}\right)+O(1)\sum_{v=1}^{m-1}\frac{p_{v+1}|\lambda_{v+1}|^k}{P_{v+1}}\\
&+ O(1)|\lambda_m|^k\\
&= O(1)\sum_{v=1}^{m-1}\frac{p_v|\lambda_v|^k}{P_v}+O(1)\sum_{v=1}^{m-1}\frac{p_{v+1}|\lambda_{v+1}|^k}{P_{v+1}}+O(1)\sum_{v=1}^{m-1}\frac{p_{v+1}|\lambda_{v+1}|^k}{P_{v+1}}+O(1)\\
&= O(1)\quad as\quad m\to\infty.
\end{aligned}$$

Therefore, we prove that $\sum_{n=1}^{\infty}\phi_n^{\delta k+k-1}|D_{n,r}|^k<\infty$ for $r=1,2,3,4$ so the proof of Theorem 3 is completed.

5 Conclusion

The present work establishes a summability factor theorem for the generalized $\phi-|B;\delta|_k$ summability method associated with positive normal matrices. Choosing $\delta=0$, $\phi_n=P_n/p_n$ and $b_{nv}=\frac{p_v}{P_n}$, the theorem yields a weighted mean matrix analogue of Theorem 2 under some additional assumptions. The present study is restricted to positive normal matrices satisfying conditions (12)–(18). Extending the obtained results to more general classes of matrices or other summability methods may be considered in future research.

References

[1] Bor, H. On $|\overline{N},p_n|_k$ summability factors of infinite series, *Comm. Fac. Sci. Univ. Ankara Ser. A_1 Math.*, V.31, N.15, 1982, pp.129–133.

[2] Bor, H. On two summability methods, *Math. Proc. Cambridge Philos. Soc.*, V.97, N.1, 1985, pp.147-149.

[3] Bor, H. On the absolute summability factors for infinite series, *Riv. Mat. Univ. Parma (4)*, V.13, 1987, pp.147–152.

[4] Bor, H. On absolute summability factors, *Analysis*, V.7, N.2, 1987, pp.185–193.

[5] Karakaş, A. On absolute matrix summability factors of infinite series, *J. Class. Anal.*, V.13, N.2, 2018, pp.133-139.

[6] Kartal, B. On an extension of absolute summability, *Konuralp J. Math.*, V.7, N.2, 2019, pp.433-437.

[7] Kartal, B. Factors for generalized matrix summability, *Erc. Unv. J. Inst. Sci and Tech.*, V.37, N.3, 2021, pp.462-467.

[8] Özarslan, H.S. A note on generalized absolute Riesz summability, *An. Ştiinţ. Univ. Al. I. Cuza Iaşi. Mat. (N.S.)*, V.60, N.1, 2014, pp.51-56.

[9] Özarslan, H.S. A new study on generalised absolute matrix summability methods, *Maejo Int. J. Sci. Technol.*, V.12, N.3, 2018, pp.199-205.

[10] Özarslan, H.S. A new factor theorem for absolute matrix summability, *Quaest. Math.*, V.42, N.6, 2019, pp.803-809.

[11] Özarslan, H.S. An application of absolute matrix summability using almost increasing and δ-quasi-monotone sequences, *Kyungpook Math. J.*, V.59, N.2, 2019, pp.233-240.

[12] Özarslan, H.S. Generalized almost increasing sequences, *Lobachevskii J. Math.*, V.42, N.1, 2021, pp.167-172.

[13] Özarslan, H.S. A new theorem on generalized absolute matrix summability, *Azerb. J. Math.*, V.13, N.1, 2023, pp.3–13.

[14] Özarslan, H.S., Karakaş, A. A new result on the almost increasing sequences, *J. Comp. Anal. Appl.*, V.22, N.6, 2017, pp.989-998.

[15] Özarslan, H.S., Kartal, B. A generalization of a theorem of Bor, *J. Inequal. Appl.*, V.179, 2017, pp.1-8.

[16] Özarslan, H.S., Kartal, B. Absolute matrix summability via almost increasing sequence, *Quaest. Math.*, V.43, N.10, 2020, pp.1477-1485.