UNDER PEER REVIEW

Applications of power increasing sequences

ABSTRACT. In the present paper, a general theorem dealing with absolute matrix summability of an infinite series has been established by using a quasi power increasing sequence instead of an almost increasing sequence. Also, some results have been obtained from this theorem.

Keywords: absolute matrix summability, almost increasing sequences, Hölder's inequality, infinite series, quasi power increasing sequences
MSC (2020): 26D15; 40D15; 40F05; 40G99.

1 Introduction

Let $\sum a_n$ be a given infinite series with the partial sums (s_n). Let (p_n) be a sequence of positive numbers such that

$$P_n = \sum_{v=0}^{n} p_v \to \infty \quad as \quad (n \to \infty), \quad (P_{-i} = p_{-i} = 0, \quad i \geq 1)\,.$$

The sequence-to-sequence transformation

$$z_n = \frac{1}{P_n} \sum_{v=0}^{n} p_v s_v$$

defines the sequence (z_n) of the Riesz mean or simply the $\left(\bar{N}, p_n\right)$ mean of the sequence (s_n), generated by the sequence of coefficients (p_n) (see [10]).

Let $B = (b_{nv})$ be a normal matrix, i.e., a lower triangular matrix of nonzero diagonal entries. Then B defines the sequence-to-sequence transformation, mapping the sequence $s = (s_n)$ to

$Bs = (B_n(s))$, where

$$B_n(s) = \sum_{v=0}^{n} b_{nv} s_v, \quad n = 0, 1, ...$$

Let (φ_n) be any sequence of positive real numbers. The series $\sum a_n$ is said to be summable $\varphi - |B, p_n|_k$, $k \geq 1$, if (see [15])

$$\sum_{n=1}^{\infty} \varphi_n^{k-1} |B_n(s) - B_{n-1}(s)|^k < \infty.$$

If we take $\varphi_n = \frac{P_n}{p_n}$, then $\varphi - |B, p_n|_k$ summability reduces to $|B, p_n|_k$ summability (see [24]). Also, if we take $\varphi_n = \frac{P_n}{p_n}$ and $b_{nv} = \frac{p_v}{P_n}$, then we get $|\bar{N}, p_n|_k$ summability (see [2]).

Given a normal matrix $B = (b_{nv})$, we associate two lower semimatrices $\bar{B} = (\bar{b}_{nv})$ and $\hat{B} = (\hat{b}_{nv})$ as follows:

$$\bar{b}_{nv} = \sum_{i=v}^{n} b_{ni}, \quad n, v = 0, 1, ... \tag{1}$$

$$\hat{b}_{00} = \bar{b}_{00} = b_{00}, \quad \hat{b}_{nv} = \bar{b}_{nv} - \bar{b}_{n-1,v}, \quad n = 1, 2, ... \tag{2}$$

and

$$B_n(s) = \sum_{v=0}^{n} b_{nv} s_v = \sum_{v=0}^{n} \bar{b}_{nv} a_v$$

$$\bar{\Delta} B_n(s) = B_n(s) - B_{n-1}(s) = \sum_{v=0}^{n} \hat{b}_{nv} a_v. \tag{3}$$

A positive sequence (g_n) is said to be almost increasing if there exist a positive increasing sequence (c_n) and two positive constant M and N such that $Mc_n \leq g_n \leq Nc_n$ (see [1]). Obviously, every increasing sequence is almost increasing. However, the converse need not be true as can be seen by taking an example, say, $b_n = ne^{(-1)^n}$. A positive sequence $X = (X_n)$ is said to be a quasi-f-power increasing sequence if there exists a constant $K = K(X, f) \geq 1$ such that $Kf_n X_n \geq f_m X_m$ for all $n \geq m \geq 1$, where $f = (f_n) = \{n^\delta (\log n)^\sigma, \ \sigma \geq 0, \ 0 < \delta < 1\}$(see [25]). If we take σ=0, then we get a quasi δ-power increasing sequence (see [13]). It should be noted that, every almost increasing sequence is a quasi δ-power increasing sequence for any nonnegative δ, but the converse need not be true as can be seen by taking the example, say $\gamma_n = n^{-\delta}$ for $\delta > 0$. A sequence (λ_n) is said to be of bounded variation, denoted by $(\lambda_n) \in \mathcal{BV}$, if $\sum_{n=1}^{\infty} |\Delta\lambda_n| = \sum_{n=1}^{\infty} |\lambda_n - \lambda_{n+1}| < \infty$.

This work considers quasi-f-power increasing sequences as the underlying structural framework. This choice enables us to extend and refine existing summability results compared with almost increasing sequences, demonstrating that weaker conditions are sufficient to obtain meaningful and strong convergence theorems.

2 Known Result

In [3], Bor has proved the following theorem for $\left|\bar{N}, p_n\right|_k$ summability factors of infinite series by using an almost increasing sequence.

Theorem 1 *Let (X_n) be an almost increasing sequence and let there be sequences (β_n) and (λ_n) such that*

$$|\Delta\lambda_n| \leq \beta_n, \tag{4}$$

$$\beta_n \to 0 \quad as \quad n \to \infty, \tag{5}$$

$$\sum_{n=1}^{\infty} n\,|\Delta\beta_n|\,X_n < \infty, \tag{6}$$

$$|\lambda_n| X_n = O(1). \tag{7}$$

If

$$\sum_{n=1}^{m} \frac{|\lambda_n|}{n} = O(1) \quad as \quad m \to \infty, \tag{8}$$

$$\sum_{n=1}^{m} \frac{1}{n}|t_n|^k = O(X_m) \quad as \quad m \to \infty,$$

and (p_n) is a sequence such that

$$\sum_{n=1}^{m} \frac{p_n}{P_n}\,|t_n|^k = O\left(X_m\right) \quad as \quad m \to \infty,$$

where (t_n) is the n-th $(C,1)$ mean of the sequence (na_n), then the series $\sum a_n\lambda_n$ is summable $\left|\bar{N}, p_n\right|_k, k \geq 1$.

3 Main Result

Many works dealing with absolute summability and absolute matrix summability factors of an infinite series have been done (see [4, 5, 7–9, 11, 12, 17–23]). The aim of this paper is to prove following theorem which generalizes Theorem 1 by applying quasi power increasing sequences.

Theorem 2 *Let $B = (b_{nv})$ be a positive normal matrix such that*

$$\bar{b}_{n0} = 1, \;\; n = 0, 1, ..., \tag{9}$$

$$b_{n-1,v} \geq b_{nv}, \;\; for \; n \geq v + 1, \tag{10}$$

$$b_{nn} = O\left(\frac{p_n}{P_n}\right). \tag{11}$$

Let (X_n) be a quasi-f-power increasing sequence for some $0 < \delta < 1$, $\sigma \geq 0$ and $(\frac{\varphi_n p_n}{P_n})$ be a non-increasing sequence. If $(\lambda_n) \in \mathcal{BV}$, the conditions (4)-(8) of Theorem 1 and

$$\sum_{n=1}^{m} \varphi_n^{k-1} \left(\frac{p_n}{P_n}\right)^{k-1} \frac{1}{n} |t_n|^k = O(X_m) \quad as \quad m \to \infty, \tag{12}$$

$$\sum_{n=1}^{m} \varphi_n^{k-1} \left(\frac{p_n}{P_n}\right)^{k} |t_n|^k = O(X_m) \quad as \quad m \to \infty \tag{13}$$

are satisfied, then the series $\sum a_n \lambda_n$ is summable $\varphi - |B, p_n|_k$, $k \geq 1$.

We need the following lemma for the proof the theorem.

Lemma 1 *([6]) Under the conditions of Theorem 2, the following conditions hold;*

$$n\beta_n X_n = O\left(1\right) \quad as \quad n \to \infty,$$

$$\sum_{n=1}^{\infty} \beta_n X_n < \infty. \tag{14}$$

4 Proof of Theorem 2

Let (I_n) denotes B-transform of the series $\sum a_n \lambda_n$. Then, by (3), we have

$$\bar{\Delta} I_n = \sum_{v=0}^{n} \hat{b}_{nv} a_v \lambda_v = \sum_{v=1}^{n} \frac{\hat{b}_{nv}\lambda_v}{v} v a_v.$$

Applying Abel's transformation to this sum, we get

$$\begin{aligned}\bar{\Delta} I_n &= \sum_{v=1}^{n-1}\Delta_v\left(\frac{\hat{b}_{nv}\lambda_v}{v}\right)\sum_{r=1}^{v} r a_r + \frac{\hat{b}_{nn}\lambda_n}{n}\sum_{r=1}^{n} r a_r\\ &= \sum_{v=1}^{n-1}\frac{v+1}{v}\Delta_v\left(\hat{b}_{nv}\right)\lambda_v t_v + \sum_{v=1}^{n-1}\frac{v+1}{v}\hat{b}_{n,v+1}\Delta\lambda_v t_v\\ &+ \sum_{v=1}^{n-1}\hat{b}_{n,v+1}\lambda_{v+1}\frac{t_v}{v} + \frac{n+1}{n}b_{nn}\lambda_n t_n\\ &= I_{n,1}+I_{n,2}+I_{n,3}+I_{n,4}.\end{aligned}$$

To complete the proof of Theorem 2, it is enough to show that

$$\sum_{n=1}^{\infty}\varphi_n^{k-1}\mid I_{n,r}\mid^k<\infty, \quad for \quad r=1,2,3,4. \tag{15}$$

First, applying Hölder's inequality with indices k and k', where $k>1$ and $\frac{1}{k}+\frac{1}{k'}=1$, we have

$$\begin{aligned}\sum_{n=2}^{m+1}\varphi_n^{k-1}\mid I_{n,1}\mid^k &= O(1)\sum_{n=2}^{m+1}\varphi_n^{k-1}\left(\sum_{v=1}^{n-1}\left|\Delta_v(\hat{b}_{nv})\right||\lambda_v||t_v|\right)^k\\ &= O(1)\sum_{n=2}^{m+1}\varphi_n^{k-1}\left(\sum_{v=1}^{n-1}\left|\Delta_v(\hat{b}_{nv})\right||\lambda_v|^k|t_v|^k\right)\times\left(\sum_{v=1}^{n-1}|\Delta_v(\hat{b}_{nv})|\right)^{k-1}.\end{aligned}$$

By (1) and (2), we have that

$$\begin{aligned}\Delta_v(\hat{b}_{nv})=\hat{b}_{nv}-\hat{b}_{n,v+1} &= \bar{b}_{nv}-\bar{b}_{n-1,v}-\bar{b}_{n,v+1}+\bar{b}_{n-1,v+1}\\ &= b_{nv}-b_{n-1,v}.\end{aligned} \tag{16}$$

Thus using (1), (9) and (10)

$$\sum_{v=1}^{n-1}|\Delta_v(\hat{b}_{nv})|=\sum_{v=1}^{n-1}(b_{n-1,v}-b_{nv})\le b_{nn}.$$

Thus

$$\begin{aligned}\sum_{n=2}^{m+1}\varphi_n^{k-1}\mid I_{n,1}\mid^k &= O(1)\sum_{n=2}^{m+1}\varphi_n^{k-1}b_{nn}^{k-1}\left(\sum_{v=1}^{n-1}\left|\Delta_v(\hat{b}_{nv})\right||\lambda_v|^k|t_v|^k\right)\\ &= O(1)\sum_{n=2}^{m+1}\left(\frac{\varphi_n p_n}{P_n}\right)^{k-1}\left(\sum_{v=1}^{n-1}\left|\Delta_v(\hat{b}_{nv})\right||\lambda_v|^k|t_v|^k\right)\\ &= O(1)\sum_{v=1}^{m}|\lambda_v|^k|t_v|^k\sum_{n=v+1}^{m+1}\left(\frac{\varphi_n p_n}{P_n}\right)^{k-1}|\Delta_v(\hat{b}_{nv})|\\ &= O(1)\sum_{v=1}^{m}\left(\frac{\varphi_v p_v}{P_v}\right)^{k-1}|\lambda_v|^k|t_v|^k\sum_{n=v+1}^{m+1}|\Delta_v(\hat{b}_{nv})|.\end{aligned}$$

Now, using (10) and (16), we obtain

$$\sum_{n=v+1}^{m+1} |\Delta_v(\hat{b}_{nv})| = \sum_{n=v+1}^{m+1} (b_{n-1,v} - b_{nv}) \leq b_{vv},$$

and so

$$\begin{aligned}
\sum_{n=2}^{m+1} \varphi_n^{k-1} \mid I_{n,1} \mid^k &= O(1) \sum_{v=1}^{m} \left(\frac{\varphi_v p_v}{P_v}\right)^{k-1} |\lambda_v|^{k-1} |\lambda_v| |t_v|^k b_{vv} \\
&= O(1) \sum_{v=1}^{m} \varphi_v^{k-1} \left(\frac{p_v}{P_v}\right)^k |\lambda_v| \, |t_v|^k \\
&= O(1) \sum_{v=1}^{m-1} \Delta |\lambda_v| \sum_{n=1}^{v} \varphi_n^{k-1} \left(\frac{p_n}{P_n}\right)^k |t_n|^k + O(1)|\lambda_m| \sum_{v=1}^{m} \varphi_v^{k-1} \left(\frac{p_v}{P_v}\right)^k |t_v|^k \\
&= O(1) \sum_{v=1}^{m-1} |\Delta \lambda_v| X_v + O(1)|\lambda_m| X_m \\
&= O(1) \sum_{n=1}^{m-1} \beta_v X_v + O(1)|\lambda_m| X_m \\
&= O(1) \quad as \quad m \to \infty,
\end{aligned}$$

by virtue of the hypotheses of Theorem 2 and Lemma 1.

By Hölder's inequality, we have

$$\begin{aligned}
\sum_{n=2}^{m+1} \varphi_n^{k-1} \mid I_{n,2} \mid^k &= O(1) \sum_{n=2}^{m+1} \varphi_n^{k-1} \left(\sum_{v=1}^{n-1} |\hat{b}_{n,v+1}| |\Delta \lambda_v| |t_v| \right)^k \\
&= O(1) \sum_{n=2}^{m+1} \varphi_n^{k-1} \left(\sum_{v=1}^{n-1} |\hat{b}_{n,v+1}| |\Delta \lambda_v| |t_v|^k \right) \times \left(\sum_{v=1}^{n-1} |\hat{b}_{n,v+1}| |\Delta \lambda_v| \right)^{k-1} \\
&= O(1) \sum_{n=2}^{m+1} \varphi_n^{k-1} b_{nn}^{k-1} \left(\sum_{v=1}^{n-1} |\hat{b}_{n,v+1}| \beta_v |t_v|^k \right) \times \left(\sum_{v=1}^{n-1} |\Delta \lambda_v| \right)^{k-1} \\
&= O(1) \sum_{n=2}^{m+1} \left(\frac{\varphi_n p_n}{P_n}\right)^{k-1} \left(\sum_{v=1}^{n-1} |\hat{b}_{n,v+1}| \beta_v |t_v|^k \right) \\
&= O(1) \sum_{v=1}^{m} \beta_v |t_v|^k \sum_{n=v+1}^{m+1} \left(\frac{\varphi_n p_n}{P_n}\right)^{k-1} |\hat{b}_{n,v+1}| \\
&= O(1) \sum_{v=1}^{m} \left(\frac{\varphi_v p_v}{P_v}\right)^{k-1} \beta_v |t_v|^k \sum_{n=v+1}^{m+1} |\hat{b}_{n,v+1}|.
\end{aligned}$$

By (1), (2), (9) and (10), we obtain

$$|\hat{b}_{n,v+1}| = \sum_{i=0}^{v}(b_{n-1,i} - b_{ni}).$$

Thus, using (1) and (9), we have

$$\sum_{n=v+1}^{m+1} |\hat{b}_{n,v+1}| = \sum_{n=v+1}^{m+1}\sum_{i=0}^{v}(b_{n-1,i} - b_{ni}) \leq 1,$$

thus

$$\begin{aligned}
\sum_{n=2}^{m+1} \varphi_n^{k-1} \mid I_{n,2} \mid^k &= O(1)\sum_{v=1}^{m}\varphi_v^{k-1}\left(\frac{p_v}{P_v}\right)^{k-1} v\beta_v \frac{1}{v}|t_v|^k \\
&= O(1)\sum_{v=1}^{m-1}\Delta(v\beta_v)\sum_{r=1}^{v}\varphi_r^{k-1}\left(\frac{p_r}{P_r}\right)^{k-1}\frac{1}{r}|t_r|^k \\
&+ O(1)m\beta_m\sum_{v=1}^{m}\varphi_v^{k-1}\left(\frac{p_v}{P_v}\right)^{k-1}\frac{1}{v}|t_v|^k \\
&= O(1)\sum_{v=1}^{m-1} v|\Delta\beta_v|X_v + O(1)\sum_{v=1}^{m-1}\beta_v X_v + O(1)m\beta_m X_m \\
&= O(1) \quad as \quad m \to \infty,
\end{aligned}$$

by virtue of the hypotheses of Theorem 2 and Lemma 1.

Also, we have

$$
\begin{aligned}
\sum_{n=2}^{m+1}\varphi_n^{k-1}|I_{n,3}|^k &\leq \sum_{n=2}^{m+1}\varphi_n^{k-1}\left(\sum_{v=1}^{n-1}|\hat{b}_{n,v+1}||\lambda_{v+1}|\frac{|t_v|}{v}\right)^k \\
&\leq \sum_{n=2}^{m+1}\varphi_n^{k-1}\left(\sum_{v=1}^{n-1}|\hat{b}_{n,v+1}||\lambda_{v+1}|\frac{|t_v|^k}{v}\right)\times\left(\sum_{v=1}^{n-1}|\hat{b}_{n,v+1}|\frac{|\lambda_{v+1}|}{v}\right)^{k-1} \\
&\leq \sum_{n=2}^{m+1}\varphi_n^{k-1}b_{nn}^{k-1}\left(\sum_{v=1}^{n-1}|\hat{b}_{n,v+1}||\lambda_{v+1}|\frac{|t_v|^k}{v}\right)\times\left(\sum_{v=1}^{n-1}\frac{|\lambda_{v+1}|}{v}\right)^{k-1} \\
&= O(1)\sum_{n=2}^{m+1}\varphi_n^{k-1}\left(\frac{p_n}{P_n}\right)^{k-1}\left(\sum_{v=1}^{n-1}|\hat{b}_{n,v+1}||\lambda_{v+1}|\frac{|t_v|^k}{v}\right) \\
&= O(1)\sum_{n=2}^{m+1}\left(\frac{\varphi_n p_n}{P_n}\right)^{k-1}\left(\sum_{v=1}^{n-1}|\hat{b}_{n,v+1}||\lambda_{v+1}|\frac{|t_v|^k}{v}\right) \\
&= O(1)\sum_{v=1}^{m}|\lambda_{v+1}|\frac{|t_v|}{v}^k\sum_{n=v+1}^{m+1}\left(\frac{\varphi_n p_n}{P_n}\right)^{k-1}|\hat{b}_{n,v+1}| \\
&= O(1)\sum_{v=1}^{m}\left(\frac{\varphi_v p_v}{P_v}\right)^{k-1}|\lambda_{v+1}|\frac{|t_v|}{v}^k\sum_{n=v+1}^{m+1}|\hat{b}_{n,v+1}| \\
&= O(1)\sum_{v=1}^{m}\varphi_v^{k-1}\left(\frac{p_v}{P_v}\right)^{k-1}|\lambda_{v+1}|\frac{|t_v|}{v}^k \\
&= O(1)\sum_{v=1}^{m-1}|\Delta\lambda_{v+1}|\sum_{r=1}^{v}\varphi_r^{k-1}\left(\frac{p_r}{P_r}\right)^{k-1}\frac{1}{r}|t_r|^k \\
&+ O(1)|\lambda_{m+1}|\sum_{v=1}^{m}\varphi_v^{k-1}\left(\frac{p_v}{P_v}\right)^{k-1}\frac{1}{v}|t_v|^k \\
&= O(1)\sum_{v=1}^{m-1}\beta_{v+1}X_{v+1}+O(1)|\lambda_{m+1}|X_{m+1} \\
&= O(1) \quad as \quad m\to\infty,
\end{aligned}
$$

by using (4), (7), (8), (11), (12) and (14).

Finally, as in $I_{n,1}$, we have

$$\begin{aligned}
\sum_{n=1}^{m} \varphi_n^{k-1} \mid I_{n,1} \mid^k &= O(1) \sum_{n=1}^{m} \varphi_n^{k-1} b_{nn}^k |\lambda_n|^k |t_n|^k \\
&= O(1) \sum_{n=1}^{m} \varphi_n^{k-1} \left(\frac{p_n}{P_n}\right)^k |\lambda_n|^{k-1} |\lambda_n| |t_n|^k \\
&= O(1) \sum_{n=1}^{m} \varphi_n^{k-1} \left(\frac{p_n}{P_n}\right)^k |\lambda_n| |t_n|^k \\
&= O(1) \quad as \quad m \to \infty,
\end{aligned}$$

by using (4), (7), (11), (13) and (14). This completes the proof of Theorem 2.

5 Conclusions

If we take $\sigma = 0$ in Theorem 2, then we get a theorem dealing with quasi δ-power increasing sequences (see [16]). If we take $\varphi_n = \frac{P_n}{p_n}$, $b_{nv} = \frac{p_v}{P_n}$ and (X_n) as an almost increasing sequence, then we get Theorem 1. If we take $\varphi_n = \frac{P_n}{p_n}$ and (X_n) as an almost increasing sequence, then we get a theorem dealing with $|B, p_n|_k$ summability (see [14]). In the last two cases, the condition $(\lambda_n) \in \mathcal{BV}$ is not needed.

Limitations

The results of the paper are theoretical and are restricted to absolute matrix summability factors of infinite series under the stated hypotheses. The applicability of the main theorem depends on several technical assumptions involving the normal matrix, the sequence ((p_n)), the sequence ((\lambda_n)), and the auxiliary sequence (X_n). The paper does not provide concrete examples illustrating the verification of these assumptions for specific series or matrices. It also does not include numerical demonstrations or applications outside the framework of summability theory. The conclusions are therefore limited to the mathematical setting developed in the manuscript. Further work may consider explicit examples, comparisons with related summability methods, or additional special cases that clarify the practical use of the theorem.

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