# SYMMETRY ANALYSIS OF $(\kappa, \mu, \nu)$-PARACONTACT MANIFOLDS ADMITTING RICCI SOLITONS

Abstract. In this paper, we investigate Ricci solitons on $(\kappa, \mu, \nu)$-paracontact manifolds by employing the Riemann, $W_0$, $W_1^*$, and $W_7$-curvature tensors. The study focuses on analyzing the conditions under which a paracontact manifold admitting a Ricci soliton exhibits Ricci pseudosymmetry and Ricci semisymmetry. By exploring the interplay between Ricci solitons and these curvature structures, we establish several characterizations and derive necessary conditions that enrich the geometric understanding of paracontact manifolds. The obtained results not only extend previous studies on curvature-restricted structures but also highlight the role of Ricci solitons in shaping the intrinsic and extrinsic geometry of paracontact spaces.

## 1. Introduction

Paracontact geometry has attracted considerable attention in recent decades due to its deep connections with both pure and applied mathematics. Paracontact geometry, first introduced by Kaneyuki, subsequently drew the interest of many researchers and became a central topic of study in this field [1]. In [2], M. Markellos and C. Tsiclios introduced new non-Sasakian $(\kappa, \mu)$-contact metric structures on the unit sphere $S^3$. Meanwhile, B. Cappelletti, Mantona, and L. Di Terlizzi demonstrated that every non-Sasakian contact $(\kappa, \mu)$-space admits a canonical paracontact metric structure [3]. Therefore, combining the results of Markellos and Tsiclios [4] with those of Cappelletti and Terlizzi, it follows that the unit sphere $S^3$ possesses a paracontact metric structure.

In particular, $(\kappa, \mu, \nu)$-paracontact manifolds, as natural generalizations of paracontact metric manifolds, provide a rich framework for studying geometric structures with additional curvature constraints. These manifolds extend the scope of classical contact and paracontact spaces, offering new perspectives in the classification of geometric structures and their curvature properties. The study of $(\kappa, \mu, \nu)$-paracontact manifolds is motivated not only by their intrinsic mathematical interest but also by their potential applications in mathematical physics, relativity theory, and the geometric analysis of differential equations.

On the other hand, Ricci solitons, introduced in the context of Hamilton's Ricci flow, have become a central topic in Riemannian and pseudo-Riemannian geometry. The geometry of Ricci solitons has attracted the attention of many mathematicians, and it became even more significant following Perelman's use of Ricci solitons to

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solve the long-standing Poincaré conjecture, posed in 1904. Since then, the geometry of Ricci solitons on various manifolds has been studied in detail and continues to be investigated to this day without losing its relevance and importance ([5]-[13]).

A Ricci soliton represents a self-similar solution to the Ricci flow and serves as a natural generalization of Einstein metrics. Beyond their fundamental role in understanding the formation of singularities in Ricci flow, Ricci solitons arise in various branches of theoretical physics, including general relativity, string theory, and cosmology. Their ability to capture the balance between the Ricci curvature and the vector field generating the flow makes them powerful tools in both the study of geometric evolution and the classification of manifolds subject to curvature constraints.

The interplay between $(\kappa, \mu, \nu)$-paracontact manifolds and Ricci solitons provides a fertile ground for exploring new geometric phenomena. In particular, examining conditions such as Ricci pseudosymmetry and Ricci semisymmetry within this setting enables us to understand how the underlying curvature tensors such as the Riemann tensor, $W_0, W_1^*$ and $W_7$ shape the structure of Ricci solitons on paracontact manifolds. These investigations not only generalize classical results but also enrich the theory of paracontact geometry by unveiling deeper links between curvature operators and soliton theory.

This paper aims to analyze Ricci solitons on $(\kappa, \mu, \nu)$-paracontact manifolds in the presence of curvature restrictions defined by various tensors. We establish characterizations of Ricci pseudosymmetric and Ricci semisymmetric structures in this context, thereby contributing to the growing body of research on curvature-restricted paracontact geometry. Moreover, the results presented here highlight the importance of Ricci solitons as a bridge between abstract differential geometry and its applications in physics and applied mathematics.

From this point onward in the paper, $(\kappa, \mu, \nu)$-paracontact metric manifolds $\Xi^{(2n+1)}$ will be denoted by ${}^{(\kappa,\mu,\nu)}\mathcal{P}\text{-}\mathcal{CMM}$ for brevity.

## 2. Preliminaries

An $(2n+1)$-dimensional differentiable manifold $\Xi$ is said to have a paracontact structure if it admits a $(1,1)-$tensor field $\phi$, a vector field $\xi$ and a 1-form $\eta$ satisfying the following conditions

$$\phi^2\omega_1 = \omega_1 - \eta(\omega_1)\xi, \tag{1}$$

for any vector field $\omega_1 \in \chi(\Xi)$ in [1],

$$\eta(\xi) = 1, \eta \circ \phi = 0, \ \phi\xi = 0.$$

An almost paracontact structure is said to be normal if and only if the $(1,2)-$type torsion tensor

$$N_\phi = [\phi, \phi] - 2d\eta \otimes \xi$$

vanishes identically, where

$$[\phi, \phi](\omega_1, \omega_2) = \phi^2[\omega_1, \omega_2] + [\phi\omega_1, \phi\omega_2] - \phi[\phi\omega_1, \omega_2] - \phi[\omega_1, \phi\omega_2].$$

An almost paracontact manifold equipped with a pseudo-Riemannian metric $g$ such that

$$g(\phi\omega_1, \phi\omega_2) = -g(\omega_1, \omega_2) + \eta(\omega_1)\eta(\omega_2), \ g(\omega_1, \xi) = \eta(\omega_1)$$

for all vector fields $\omega_1, \omega_2 \in \chi(\Xi)$, is called an almost paracontact metric manifold, where the signature of $g$ is $(n+1, n)$. An almost paracontact structure is said to be a paracontact structure if $g(\omega_1, \phi\omega_2) = d\eta(\omega_1, \omega_2)$ with the associated metric $g$ [14]. Let $h$ be $(1,1)$ tensor field defined by $h = \frac{1}{2}L_\xi \phi$, where $L$ denotes the Lie derivative. Then $h$ is symmetric and satisfies the conditions

$$h\phi = -\phi h, h\xi = 0, Tr.h = Tr.\phi h = 0.$$

Let $\nabla$ be the Levi-Civita connection of $g$. So it is clear that

$$\widetilde{\nabla}_{\omega_1}\xi = -\phi\omega_1 + \phi h\omega_1 \tag{2}$$

for any $\omega_1 \in \chi(\Xi)$ [1]. For a paracontact metric manifold $\Xi$, if $\xi$ is a Killing vector field or equivalently $h = 0$, then it is called a $K-$paracontact manifold.

An almost paracontact manifold is said to be para-Sasakian if and only if the following condition holds

$$(\widetilde{\nabla}_{\omega_1}\phi)\omega_2 = -g(\omega_1, \omega_2)\xi + \eta(\omega_2)\omega_1,$$

for all $\omega_1, \omega_2 \in \chi(\Xi)$ [14]. A normal paracontact metric manifold is para-Sasakian and satisfies

$$R(\omega_1, \omega_2)\xi = -\eta(\omega_2)\omega_1 + \eta(\omega_1)\omega_2$$

for all $\omega_1, \omega_2 \in \chi(\Xi)$. But this is not a sufficient condition for a paracontact manifold to be para-Sasakian. It is clear that every para-Sasakian manifold is $K-$paracontact. But the converse is not always true [15].

A paracontact metric manifold is said to be a $(\kappa, \mu)-$paracontact manifold if the curvature tensor $\widetilde{R}$ is of form

$$\widetilde{R}(\omega_1, \omega_2)\xi = \kappa\left[\eta(\omega_2)\omega_1 - \eta(\omega_1)\omega_2\right] + \mu\left[\eta(\omega_2)h\omega_1 - \eta(\omega_1)h\omega_2\right]$$

for all $\omega_1, \omega_2 \in \chi(\Xi)$, where $\kappa$ and $\mu$ are real constants.

A $(2n+1)-$dimensional ${}^{(\kappa,\mu,\nu)}\mathcal{P}\text{-}\mathcal{CMM}$ is a paracontact metric manifold for which the curvature tensor field satisfies

$$\begin{aligned} \widetilde{R}(\omega_1, \omega_2)\xi &= \kappa\{\eta(\omega_2)\omega_1 - \eta(\omega_1)\omega_2\} + \mu\{\eta(\omega_2)h\omega_1 - \eta(\omega_1)h\omega_2\} \\ &+\nu\{\eta(\omega_2)\phi h\omega_1 - \eta(\omega_1)\phi h\omega_2\}, \end{aligned} \tag{3}$$

for all $\omega_1, \omega_2 \in \chi(\Xi)$, where $\kappa, \mu, \nu$ are smooth functions.

**Example 1.** *We consider the 3-dimensional manifold* $\Xi = \{(x,y,z) \in \mathbb{R}^3,\ z \neq 0\}$, *where* $(x,y,z)$ *are standart coordinates of* $\mathbb{R}^3$. *The vector fields*

$$e_1 = 4x^3\frac{\partial}{\partial x} + \frac{5}{4}z^4\frac{\partial}{\partial y} + \frac{\partial}{\partial z},$$

$$e_2 = \frac{\partial}{\partial y}, e_3 = \frac{\partial}{\partial z}.$$

*Let* $g$ *be the Riemannian metric defined by*

$$g(e_1, e_2) = g(e_1, e_3) = g(e_2, e_3) = 0,$$

$$g(e_1, e_1) = g(e_2, e_2) = 1, g(e_3, e_3) = -1$$

*Let* $\eta$ *be the 1-form defined by* $\eta(\omega_1) = g(\omega_1, e_2)$ *for any* $\omega_1 \in \chi(\Xi)$. *Let* $\phi$ *be the (1,1) tensor field defined by*

$$\phi(e_2) = 0, \qquad \phi(e_3) = -e_1, \qquad \phi(e_1) = -e_3.$$

*Let* $\nabla$ *be the Levi-Civita connection with respect to the metric tensor* $g$. *Then we get*

$$[e_3, e_1] = 5z^3e_2 \quad [e_1, e_2] = 0, \qquad [e_2, e_3] = 0.$$

*Then we have*

$$\eta(e_2) = g(e_2, e_2) = 1, \quad \phi^2\omega_1 = \omega_1 - \eta(\omega_1)e_2,$$

$$g(\phi\omega_1, \phi\omega_2) = -g(\omega_1, \omega_2) + \eta(\omega_1)\eta(\omega_2),$$

*for any* $\omega_1, \omega_2 \in \chi(\Xi)$. *Hence,* $(\phi, \xi, \eta, g)$ *defines a* $(\kappa, \mu, \nu)-$*paracontact metric structure on* $\Xi$ *for* $e_2 = \xi$.

*The Levi-Civita connection* $\nabla$ *of the metric* $g$ *is given by the Koszul's formula*

$$2g(\nabla_{\omega_1}\omega_2, \omega_3) = \omega_1 g(\omega_2, \omega_3) + \omega_2 g(\omega_3, \omega_1) - \omega_3 g(\omega_1, \omega_2)$$

$$-g(\omega_1, [\omega_2, \omega_3]) - g(\omega_2, [\omega_1, \omega_3]) + g(\omega_3, [\omega_1, \omega_2]).$$

*Using the above formula, we obtain*

$$\nabla_{e_1}e_1 = 0, \qquad \nabla_{e_2}e_1 = -\tfrac{5}{2}z^3 e_3, \quad \nabla_{e_3}e_1 = \tfrac{5}{2}z^3 e_2,$$

$$\nabla_{e_1}e_2 = -\tfrac{5}{2}z^3 e_3, \quad \nabla_{e_2}e_2 = 0, \qquad \nabla_{e_3}e_2 = -\tfrac{5}{2}z^3 e_1,$$

$$\nabla_{e_1}e_3 = -\tfrac{5}{2}z^3 e_2, \quad \nabla_{e_2}e_3 = -\tfrac{5}{2}z^3 e_1, \quad \nabla_{e_3}e_3 = 0.$$

*Comparing the above relations with* $\nabla_{\omega_1}e_2 = -\phi\omega_1 + \phi h\omega_1$, *we get*

$$he_1 = -(\frac{5}{2}z^3 + 1)e_2, \qquad he_3 = -(\frac{5}{2}z^3 + 1)e_3, \qquad he_2 = 0.$$

*Using the formula* $R(\omega_1, \omega_2)\omega_3 = \nabla_{\omega_1}\nabla_{\omega_2}\omega_3 - \nabla_{\omega_2}\nabla_{\omega_1}\omega_3 - \nabla_{[\omega_1,\omega_2]}\omega_3$, *we calculate the following:*

$$R(e_2, e_1)e_2 = \left[(\tfrac{5}{2}z^3 + 1)^2 - 1\right]\{\eta(e_1)e_2 - \eta(e_2)e_1\}$$

$$+5z^3\{\eta(e_1)he_2 - \eta(e_2)he_1\}$$

$$+0\{\eta(e_1)\phi he_2 - \eta(e_2)\phi he_1\}$$

$$= \tfrac{25}{4}z^6 e_1.$$

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$$+0\{\eta(e_3)\phi he_1 - \eta(e_1)\phi he_3\}$$

$$= 0.$$

*By the above expressions of the curvature tensor and using* (3)*, we conclude that* $\Xi$ *is an* ${}^{(\kappa,\mu,\nu)}\mathcal{P}$-$\mathcal{CMM}$ *with*

$$\kappa=\left[(\frac{5}{2}z^3+1)^2-1\right],$$

$$\mu=5z^3$$

*and*

$$\nu=0.$$

**Lemma 1.** *Let* $\Xi^{2n+1}$ *be an* ${}^{(\kappa,\mu,\nu)}\mathcal{P}$-$\mathcal{CMM}$*. Then the following identities hold.*

$$h^2=(1+\kappa)\phi^2,\ \textit{for}\ \kappa\neq-1, \tag{4}$$

$$\xi(\kappa)=-2\nu(1+\kappa),$$

$$Q\xi=2n\kappa\xi,$$

$$(\widetilde{\nabla}_{\omega_1}\phi)\omega_2=-g(\omega_1-h\omega_1,\omega_2)\xi+\eta(\omega_2)(\omega_1-h\omega_1),$$

$$S(\omega_1,\xi)=2n\kappa\eta(\omega_1),$$

$$\widetilde{R}(\xi,\omega_1)\omega_2=\kappa\{g(\omega_1,\omega_2)\xi-\eta(\omega_2)\omega_1\}+\mu\{g(h\omega_1,\omega_2)\xi-\eta(\omega_2)h\omega_1\}$$

$$+\nu\{g(\phi h\omega_1,\omega_2)\xi-\eta(\omega_2)\phi h\omega_1\},$$

$$\eta(R(\omega_1,\omega_2)\omega_3)=\kappa g(\eta(\omega_1)\omega_2-\eta(\omega_2)\omega_1,\omega_3)$$

$$+\mu g(\eta(\omega_1)h\omega_2-\eta(\omega_2)h\omega_1,\omega_3)$$

$$+\nu g(\eta(\omega_1)\phi h\omega_2-\eta(\omega_2)\phi h\omega_1,\omega_3).$$

*for any vector fields* $\omega_1,\omega_2\in\chi(\Xi)$*, where* $S$ *and* $Q$ *denote the Ricci tensor and Ricci operator defined by* $S(\omega_1,\omega_2)=g(Q\omega_1,\omega_2)$*.*

A Ricci soliton on a Riemannian manifold is defined as a triple $(g,\xi,\lambda)$ on $\Xi^{2n+1}$ satisfying

$$L_\xi g+2S+2\lambda g=0,$$

where $L_\xi$ is the Lie derivative operator along the vector field $\xi$ and $\lambda$ is a real constant. We note that if $\xi$ is a Killing vector field, then the Ricci soliton reduces to an Einstein metric $(g,\lambda)$.

## 3. Ricci-Pseudosymmetric $(\kappa,\mu,\nu)$-Paracontact Manifolds

Now let $(g,\xi,\lambda)$ be Ricci soliton on ${}^{(\kappa,\mu,\nu)}\mathcal{P}$-$\mathcal{CMM}$. Then we have

$$\begin{aligned}(l_\xi g)(\omega_1,\omega_2)&=g(\nabla_{\omega_1}\xi,\omega_2)+g(\nabla_{\omega_2}\xi,\omega_1)\\&=g(-\phi\omega_1+\phi h\omega_1,\omega_2)+g(-\phi\omega_2+\phi h\omega_2,\omega_1)\\&=-g(\phi\omega_1,\omega_2)+g(\phi h\omega_1,\omega_2)-g(\phi\omega_2,\omega_1)+g(\phi h\omega_2,\omega_1)\\&=2g(\phi h\omega_2,\omega_1),\end{aligned}\tag{5}$$

for all $\omega_1,\omega_2\in\chi(\Xi)$. Thus, in an ${}^{(\kappa,\mu,\nu)}\mathcal{P}$-$\mathcal{CMM}$, from (2) and (5), we have

$$g(\phi h\omega_2,\omega_1)+S(\omega_1,\omega_2)+\lambda g(\omega_1,\omega_2)=0. \tag{6}$$

For $\omega_2 = \xi$, this implies that

$$S(\omega_1, \xi) = -\lambda\eta(\omega_1). \tag{7}$$

Taking into account of (7), we arrive at

$$\lambda = -2n\kappa.$$

On a semi-Riemannian manifold $(\Xi, g)$, for a $(0,k)$-type tensor field $T$ and a $(0,2)$-type tensor field $A$, the $(0,k+2)$-type tensor field $Q(A,T)$ is defined as

$$\begin{aligned} Q(A,T)(X_1, X_2, ..., X_k; \omega_1, \omega_2) &= -T((\omega_1 \wedge_A \omega_2)X_1, X_2, ..., X_k) \\ -T(X_1, (\omega_1 \wedge_A \omega_2)X_2, X_3, ..., X_k) &- ... - T(X_1, X_2, ..., (\omega_1 \wedge_A \omega_2)X_k) \end{aligned} \tag{8}$$

for all $X_1, X_2, ..., X_k, \omega_1, \omega_2, \omega_3 \in \Gamma(T\Xi)$, where

$$(\omega_1 \wedge_A \omega_2)\omega_3 = A(\omega_2, \omega_3)\omega_1 - A(\omega_1, \omega_3)\omega_2.$$

**Definition 1.** *Let $\Xi^{2n+1}$ be an $(2n+1)-$dimensional $^{(\kappa,\mu,\nu)}\mathcal{P}$-$\mathcal{CMM}$. If $R \cdot S$ and $Q\,(g, S)$ are linearly dependent, then the $\Xi^{2n+1}$ is said to be **Ricci pseudosymmetric**.*

In this case, there exists a function $L_1$ on $\Xi^{2n+1}$ such that

$$R \cdot S = L_1 Q\,(g, S)\,.$$

In particular, if $L_1 = 0$, the manifold $\Xi^{2n+1}$ is said to be **Ricci semisymmetric**.

Let us now investigate the Ricci pseudosymmetric case of the $(2n+1)-$dimensional $^{(\kappa,\mu,\nu)}\mathcal{P}$-$\mathcal{CMM}$.

**Theorem 1.** *Let $\Xi^{2n+1}$ be a $^{(\kappa,\mu,\nu)}\mathcal{P}$-$\mathcal{CMM}$ and $(g, \xi, \lambda)$ be Ricci soliton on $\Xi^{2n+1}$. If $\Xi^{2n+1}$ is a Ricci pseudosymmetric $^{(\kappa,\mu,\nu)}\mathcal{P}$-$\mathcal{CMM}$, then the function $L_1$ satisfies*

$$L_1 = \kappa, \;\; \mu = \pm\sqrt{\nu}.$$

*Proof.* Let us assume that $^{(\kappa,\mu,\nu)}\mathcal{P}$-$\mathcal{CMM}$ be Ricci pseudosymmetric and $(g, \xi, \lambda)$ is an almost Ricci soliton on the $^{(\kappa,\mu,\nu)}\mathcal{P}$-$\mathcal{CMM}$. This mean

$$(R(\omega_1, \omega_2) \cdot S)(\omega_4, \omega_5) = L_1 Q(g, S)(\omega_4, \omega_5; \omega_1, \omega_2),$$

for all $\omega_1, \omega_2, \omega_4, \omega_5 \in \chi(\Xi)$, which implies that

$$S(R(\omega_1, \omega_2)\omega_4, \omega_5) + S(\omega_4, R(\omega_1, \omega_2)\omega_5) = L_1\{S((\omega_1 \wedge_g \omega_2)\omega_4, \omega_5) + S(\omega_4, (\omega_1 \wedge_g \omega_2)\omega_5)\}. \tag{9}$$

Substituting $\omega_5 = \xi$ in (9), we have

$$\begin{aligned} &S(R(\omega_1, \omega_2)\omega_4, \xi) + S(\omega_4, R(\omega_1, \omega_2)\xi) \\ &= L_1\{S(g(\omega_2, \omega_4)\omega_1 - g(\omega_1, \omega_4)\omega_2, \xi) \\ &+ S(\omega_4, \eta(\omega_2)\omega_1 - \eta(\omega_1)\omega_2)\}. \end{aligned} \tag{10}$$

Substituting (7) into (10), we obtain

$$\begin{aligned} &-\lambda\eta(R(\omega_1, \omega_2)\omega_4) + S(\omega_4, \kappa[\eta(\omega_2)\omega_1 - \eta(\omega_1)\omega_2] + \mu[\eta(\omega_2)h\omega_1 \\ &-\eta(\omega_1)h\omega_2] + \nu[\eta(\omega_2)\phi h\omega_1 - \eta(\omega_1)\phi h\omega_2]) \\ &= L_1\{-\lambda g(\omega_4, \eta(\omega_1)\omega_2 - \eta(\omega_2)\omega_1) + S(\omega_4, \eta(\omega_2)\omega_1 - \eta(\omega_1)\omega_2)\}. \end{aligned} \tag{11}$$

Also, taking into account (8) and (11), we arrive at

$$\begin{aligned}
&-\lambda(\kappa g(\eta(\omega_1)\omega_2-\eta(\omega_2)\omega_1,\omega_3)+\mu g(\eta(\omega_1)h\omega_2-\eta(\omega_2)h\omega_1,\omega_3)\\
&+\nu g(\eta(\omega_1)\phi h\omega_2-\eta(\omega_2)\phi h\omega_1,\omega_3))\\
&=-L_1\lambda g(\eta(\omega_1)\omega_2-\eta(\omega_2)\omega_1,\omega_4)+L_1 g(\eta(\omega_1)\omega_2-\eta(\omega_2)\omega_1,\phi h\omega_4)\\
&+L_1\lambda g(\eta(\omega_1)\omega_2-\eta(\omega_2)\omega_1,\omega_4).
\end{aligned}\tag{12}$$

From (12) , using (6) , we infer

$$\begin{aligned}
&-\kappa g(\omega_1,\phi h\omega_4)-\mu g(\phi h\omega_4,h\omega_1)-\nu g(\phi h\omega_4,\phi h\omega_1)\\
&+L_1 g(\phi h\omega_4,\omega_1)=0.
\end{aligned}$$

Consequently, we have

$$(L_1-\kappa)\omega_1-\mu h\omega_1-\nu\phi h\omega_1=0, \tag{13}$$

which implies that

$$(L_1-\kappa)=0.$$

From (13), we get

$$\mu h\omega_1+\nu\phi h\omega_1=0. \tag{14}$$

By virtue of (1) and (14), we obtain

$$[\mu^2-\nu^2]h\omega_1=0$$

This completes the proof. □

**Corollary 1.** *Let $\Xi^{2n+1}$ is a Ricci semisymmetric $^{(\kappa,\mu,\nu)}\mathcal{P}$-$\mathcal{CMM}$ and $(g,\xi,\lambda)$ be a Ricci soliton on $\Xi^{2n+1}$. Then, we have*

$$\kappa=0,\mu\pm\sqrt{\nu}=0.$$

**Remark 1.** *If the manifold chosen in the above example is Ricci pseudosymmetric, then we have*

$$L_1=\left(\frac{5}{2}z^3+1\right)^2-1.$$

For a $(2n+1)$-dimensional semi-Riemannian manifold $\Xi$, the $W_0$-curvature tensor is defined as

$$W_0\left(\omega_1,\omega_2\right)\omega_3=R(\omega_1,\omega_2)\omega_3-\frac{1}{2n}\left[S(\omega_2,\omega_3)\omega_1-g(\omega_1,\omega_3)Q\omega_2\right]. \tag{15}$$

For a $(2n+1)$ −dimensional $^{(\kappa,\mu,\nu)}\mathcal{P}$-$\mathcal{CMM}$, if we choose $\omega_3=\xi$ in (15), we can write

$$\begin{aligned}
W_0(\omega_1,\omega_2)\xi&=-\kappa\eta(\omega_1)\omega_2+\tfrac{1}{2n}\eta(\omega_1)Q\omega_2\\
&+\mu(\eta(\omega_2)h\omega_1-\eta(\omega_1)h\omega_2)\\
&+\nu(\eta(\omega_2)\phi h\omega_1-\eta(\omega_1)\phi h\omega_2),
\end{aligned}$$

and with the help of (15), we get

$$\eta(W_0(\omega_1,\omega_2)\omega_3) = g(\kappa\eta(\omega_1)\omega_2 - \tfrac{1}{2n}\eta(\omega_1)Q\omega_2,\omega_3)$$
$$(16) \qquad +\mu g(\eta(\omega_1)h\omega_2 - \eta(\omega_2)h\omega_1,\omega_3) + \nu g(\eta(\omega_1)\phi h\omega_2$$
$$-\eta(\omega_2)\phi h\omega_1,\omega_3).$$

**Theorem 2.** *Let $\Xi^{2n+1}$ be a $^{(\kappa,\mu,\nu)}\mathcal{P}$-$\mathcal{CMM}$ and $(g,\xi,\lambda)$ be a Ricci soliton on $\Xi^{2n+1}$. If $\Xi^{2n+1}$ is a $W_0-$Ricci pseudosymmetric $^{(\kappa,\mu,\nu)}\mathcal{P}$-$\mathcal{CMM}$, then the function $L_2$ satisfies*

$$L_2 = -\nu(1+\kappa), \mu(1+\kappa) = 0.$$

*Proof.* Let us assume that $^{(\kappa,\mu,\nu)}\mathcal{P}$-$\mathcal{CMM}$ be $W_0$-Ricci pseudosymmetric and $(g,\xi,\lambda)$ is an almost Ricci soliton on the $^{(\kappa,\mu,\nu)}\mathcal{P}$-$\mathcal{CMM}$. This mean

$$(W_0(\omega_1,\omega_2)\cdot S)(\omega_4,\omega_3) = L_2 Q(g,S)(\omega_4,\omega_3;\omega_1,\omega_2),$$

for all $\omega_1,\omega_2,\omega_4,\omega_3 \in \chi(\Xi)$, which implies that

$$S(W_0(\omega_1,\omega_2)\omega_4,\omega_3) + S(\omega_4,W_0(\omega_1,\omega_2)\omega_3)$$
$$(17) \qquad = L_2\{S((\omega_1 \wedge_g \omega_2)\omega_4,\omega_3) + S(\omega_4,(\omega_1 \wedge_g \omega_2)\omega_3)\}.$$

Substituting $\omega_3 = \xi$ in (17), we observe

$$S(W_0(\omega_1,\omega_2)\omega_4,\xi) + S(\omega_4,W_0(\omega_1,\omega_2)\xi)$$
$$(18) \qquad = L_2\{S(g(\omega_2,\omega_4)\omega_1 - g(\omega_1,\omega_4)\omega_2,\xi)$$
$$+S(\omega_4,\eta(\omega_2)\omega_1 - \eta(\omega_1)\omega_2)\}.$$

Substituting (7) into (18), we obtain

$$-\lambda\eta(W_0(\omega_1,\omega_2)\omega_4) + S(\omega_4,-\kappa\eta(\omega_1)\omega_2 + \eta(\omega_1)Q\omega_2$$
$$+\mu(\eta(\omega_2)h\omega_1 - \eta(\omega_1)h\omega_2) + \nu(\eta(\omega_2)\phi h\omega_1 - \eta(\omega_1)\phi h\omega_2))$$
$$= L_2\{-\lambda g(\omega_4,\eta(\omega_1)\omega_2 - \eta(\omega_2)\omega_1) + S(\omega_4,\eta(\omega_2)\omega_1 - \eta(\omega_1)\omega_2)\}.$$

Also, taking into account (6) and (16), we can infer

$$-\lambda g(\omega_4,\kappa\eta(\omega_1)\omega_2 - \tfrac{1}{2n}\eta(\omega_1)Q\omega_2 - \lambda\mu g(\omega_4,\eta(\omega_1)h\omega_2 - \eta(\omega_2)h\omega_1)$$
$$-\lambda\nu g(\omega_4,\eta(\omega_1)\phi h\omega_2 - \eta(\omega_2)\phi h\omega_1) - g(\phi h\omega_4,-\kappa\eta(\omega_1)\omega_2 + \tfrac{1}{2n}\eta(\omega_1)Q\omega_2)$$
$$(19) \qquad -\lambda g(\omega_4,-\kappa\eta(\omega_1)\omega_2 + \tfrac{1}{2n}\eta(\omega_1)Q\omega_2) - \mu g(\phi h\omega_4,\eta(\omega_2)h\omega_1 - \eta(\omega_1)h\omega_2)$$
$$-\mu\lambda g(\omega_4,\eta(\omega_2)h\omega_1 - \eta(\omega_1)h\omega_2) - \nu g(\phi h\omega_4,\eta(\omega_2)\phi h\omega_1 - \eta(\omega_1)\phi h\omega_2)$$
$$-\nu\lambda g(\omega_4,\eta(\omega_2)\phi h\omega_1 - \eta(\omega_1)\phi h\omega_2) + L_2 g(\eta(\omega_2)\omega_1 - \eta(\omega_1)\omega_2,\phi h\omega_4) = 0.$$

In (19), setting $\omega_2 = \xi$ and using (4), we arrive at

$$-\mu g(\phi h\omega_4,h\omega_1) - \nu g(\phi h\omega_4,\phi h\omega_1) + L_2 g(\omega_1,\phi h\omega_4) = 0.$$

Here, by using (1) and (4), we obtain

$$\mu(1+\kappa)g(\phi\omega_4,\omega_1)-\nu(1+\kappa)g(\phi\omega_1,\phi\omega_4)-L_2 g(\phi\omega_1,\phi\omega_4)=0,$$

which implies that

$$\mu(1+\kappa)\omega_1-\nu(1+\kappa)\phi\omega_1-L_2\phi\omega_1=0.$$

Thus, this completes the proof. □

**Corollary 2.** *Let $\Xi^{2n+1}$ is a $W_0$-Ricci semisymmetric $^{(\kappa,\mu,\nu)}\mathcal{P}$-$\mathcal{CMM}$ and $(g,\xi,\lambda)$ be a Ricci soliton on $\Xi^{2n+1}$. Then, we have*

$$-\upsilon\,(1+\kappa)=0 \text{ and } \mu\,(1+\kappa)=0.$$

**Remark 2.** *If the manifold chosen in the above example is $W_0-$Ricci pseudosymmetric, then we have*

$$L_2=0 \text{ and } 5z^3\left[\left(\frac{5}{2}z^3+1\right)^2-1\right]=0.$$

For a $(2n+1)$-dimensional semi-Riemannian manifold $\Xi$, the $W_0$-curvature tensor is defined as

$$W_7\,(\omega_1,\omega_2)\,\omega_3=R(\omega_1,\omega_2)\omega_3-\frac{1}{2n}\left[S(\omega_2,\omega_3)\omega_1-g(\omega_2,\omega_3)Q\omega_1\right]. \tag{20}$$

For a $(2n+1)$-dimensional $^{(\kappa,\mu,\nu)}\mathcal{P}$-$\mathcal{CMM}$, if we choose $\omega_3=\xi$ in (20), we can write

$$\begin{aligned} W_7(\omega_1,\omega_2)\xi &= -\kappa\eta(\omega_1)\omega_2+\tfrac{1}{2n}\eta(\omega_2)Q\omega_1+\mu(\eta(\omega_2)h\omega_1 \\ &-\eta(\omega_1)h\omega_2)+\nu(\eta(\omega_2)\phi h\omega_1-\eta(\omega_1)\phi h\omega_2), \end{aligned} \tag{21}$$

and with the help of (21), we get

$$\begin{aligned} \eta(W_7(\omega_1,\omega_2)\omega_3) &= g(\kappa\eta(\omega_1)\omega_2-\tfrac{1}{2n}\eta(\omega_2)Q\omega_1,\omega_3)+\mu g(\eta(\omega_1)h\omega_2 \\ &-\eta(\omega_2)h\omega_1,\omega_3)+\nu(\eta(\omega_1)\phi h\omega_2-\eta(\omega_2)\phi h\omega_1,\omega_3). \end{aligned}$$

**Theorem 3.** *Let $\Xi^{2n+1}$ be a $^{(\kappa,\mu,\nu)}\mathcal{P}$-$\mathcal{CMM}$ and $(g,\xi,\lambda)$ be a Ricci soliton on $\Xi^{2n+1}$. If $\Xi^{2n+1}$ is a $W_7-$Ricci pseudosymmetric $^{(\kappa,\mu,\nu)}\mathcal{P}$-$\mathcal{CMM}$, then the function $L_3$ satisfies*

$$(L_3-\kappa)^2+\nu^2(1+\kappa)=0.$$

*Proof.* Let us assume that $^{(\kappa,\mu,\nu)}\mathcal{P}$-$\mathcal{CMM}$ be $W_7-$Ricci pseudosymmetric and $(g,\xi,\lambda)$ is an almost Ricci soliton on the $^{(\kappa,\mu,\nu)}\mathcal{P}$-$\mathcal{CMM}$. This mean

$$(W_7(\omega_1,\omega_2)\cdot S)(\omega_4,\omega_3)=L_3 Q(g,S)(\omega_4,\omega_3;\omega_1,\omega_2),$$

for all $\omega_1,\omega_2,\omega_4,\omega_3\in\chi(\Xi)$, which implies that

$$\begin{aligned} &L_3\{S((\omega_1\wedge_g\omega_2)\omega_4,\omega_3)+S(\omega_4,(\omega_1\wedge_g\omega_2)\omega_3)\} \\ &=S(W_7(\omega_1,\omega_2)\omega_4,\omega_3)+S(\omega_4,W_7(\omega_1,\omega_2)\omega_3). \end{aligned} \tag{22}$$

Substituting $\omega_3=\xi$ in (22), we observe

$$
\begin{aligned}
&S(W_7(\omega_1,\omega_2)\omega_4,\xi)+S(\omega_4,W_7(\omega_1,\omega_2)\xi)\\
&=L_3\{S(g(\omega_2,\omega_4)\omega_1-g(\omega_1,\omega_4)\omega_2,\xi)\\
&+S(\omega_4,\eta(\omega_2)\omega_1-\eta(\omega_1)\omega_2)\}.
\end{aligned}
\tag{23}
$$

Substituting (6) into (23), we obtain

$$
\begin{aligned}
&-\lambda\eta(W_7(\omega_1,\omega_2)\omega_4)+S(\omega_4,-\kappa\eta(\omega_1)\omega_2-\tfrac{1}{2n}\eta(\omega_2)Q\omega_1\\
&+\mu(\eta(\omega_2)h\omega_1-\eta(\omega_1)h\omega_2)+\nu(\eta(\omega_2)\phi h\omega_1-\eta(\omega_1)\phi h\omega_2))\\
&=L_3\{-\lambda g(\omega_4,\eta(\omega_1)\omega_2-\eta(\omega_2)\omega_1)+S(\omega_4,\eta(\omega_2)\omega_1-\eta(\omega_1)\omega_2)\}.
\end{aligned}
$$

Also, taking into account (6) and (21), we can infer

$$
\begin{aligned}
&-\lambda g(\omega_4,\kappa\eta(\omega_1)\omega_2-\tfrac{1}{2n}\eta(\omega_2)Q\omega_1)-\lambda\mu g(\omega_4,\eta(\omega_1)h\omega_2-\eta(\omega_2)h\omega_1)\\
&-\lambda\nu g(\omega_4,\eta(\omega_1)\phi h\omega_2-\eta(\omega_2)\phi h\omega_1)-g(\phi h\omega_4,-\kappa\eta(\omega_1)\omega_2+\tfrac{1}{2n}\eta(\omega_2)Q\omega_1)\\
&-\lambda g(\omega_4,-\kappa\eta(\omega_1)\omega_2+\tfrac{1}{2n}\eta(\omega_2)Q\omega_1)-\mu g(\phi h\omega_4,\eta(\omega_2)h\omega_1-\eta(\omega_1)h\omega_2)\\
&-\mu\lambda g(\omega_4,\eta(\omega_2)h\omega_1-\eta(\omega_1)h\omega_2)-\nu g(\phi h\omega_4,\eta(\omega_2)\phi h\omega_1-\eta(\omega_1)\phi h\omega_2)\\
&-\nu\lambda g(\omega_4,\eta(\omega_2)\phi h\omega_1-\eta(\omega_1)\phi h\omega_2)+L_3 g(\eta(\omega_2)\omega_1-\eta(\omega_1)\omega_2,\phi h\omega_4)=0.
\end{aligned}
\tag{24}
$$

Here setting $\omega_1=\xi$ and using (4), we arrive at

$$
\begin{aligned}
&(L_3-\kappa)g(h\omega_2,\phi\omega_4)-\mu(1+\kappa)g(\omega_2,\phi\omega_4)\\
&+\nu(1+\kappa)g(\phi\omega_2,\phi\omega_4)=0,
\end{aligned}
\tag{25}
$$

consequently, we arrive at

$$-\mu(1+\kappa)\omega_2+(L_3-\kappa)h\omega_2+\nu(1+\kappa)\phi\omega_2=0.$$

which implies that

$$\mu(1+\kappa)=0.$$

With the help of (24) and (25), we conclude that

$$(L_3-\kappa)^2+\nu^2(1+\kappa)=0.$$

Thus, the proof is completed. □

**Corollary 3.** *Let $\Xi^{2n+1}$ is a $W_7-$Ricci semisymmetric ${}^{(\kappa,\mu,\nu)}\mathcal{P}$-$\mathcal{CMM}$ and $(g,\xi,\lambda)$ be a Ricci soliton on $\Xi^{2n+1}$. Then, we have*

$$\pm\kappa-v\,(1+\kappa)=0.$$

**Remark 3.** *If the manifold chosen in the above example is $W_7-$Ricci pseudosymmetric, then we have*

$$L_3=\pm\sqrt{\left(\frac{5}{2}z^3+1\right)^2-1}.$$

For a $(2n+1)$-dimensional semi-Riemannian manifold $\Xi$, the $W_1^*$-curvature tensor is defined as

$$W_1^*(\omega_1,\omega_2)\omega_3 = R(\omega_1,\omega_2)\omega_3 + \frac{1}{2n}\{S(\omega_2,\omega_3)\omega_1 - S(\omega_1,\omega_3)\omega_2\}. \tag{26}$$

For a $(2n+1)$-dimensional ${}^{(\kappa,\mu,\nu)}\mathcal{P}$-$\mathcal{CMM}$, if we choose $\omega_3 = \xi$ in (26), we can write

$$W_1^*(\omega_1,\omega_2)\xi = \mu(\eta(\omega_2)h\omega_1 - \eta(\omega_1)h\omega_2) + \nu(\eta(\omega_2)\phi h\omega_1 - \eta(\omega_1)\phi h\omega_2) \tag{27}$$

and with the help of (27) , we get

$$\eta(W_1^*(\omega_1,\omega_2)\omega_3) = \mu g(\eta(\omega_1)h\omega_2 - \eta(\omega_2)h\omega_1,\omega_3) + \nu g(\eta(\omega_1)\phi h\omega_2 - \eta(\omega_2)\phi h\omega_1,\omega_3). \tag{28}$$

**Theorem 4.** *Let $\Xi^{2n+1}$ be a ${}^{(\kappa,\mu,\nu)}\mathcal{P}$-$\mathcal{CMM}$ and $(g,\xi,\lambda)$ be a Ricci soliton on $\Xi^{2n+1}$. If $\Xi^{2n+1}$ is a $W_1^*-$Ricci pseudosymmetric ${}^{(\kappa,\mu,\nu)}\mathcal{P}$-$\mathcal{CMM}$, then the function $L_4$ satisfies*

$$(L_4^2 + \nu^2)(1+\kappa) = 0.$$

*Proof.* Let us assume that the ${}^{(\kappa,\mu,\nu)}\mathcal{P}$-$\mathcal{CMM}$ be $W_7$-Ricci pseudosymmetric and $(g,\xi,\lambda)$ is an almost Ricci soliton on the ${}^{(\kappa,\mu,\nu)}\mathcal{P}$-$\mathcal{CMM}$ . This mean

$$(W_1^*(\omega_1,\omega_2)\cdot S)(\omega_4,\omega_3) = L_4 Q(g,S)(\omega_4,\omega_3;\omega_1,\omega_2),$$

for all $\omega_1,\omega_2,\omega_4,\omega_3 \in \chi(\Xi)$, which implies that

$$\begin{aligned} &S(W_1^*(\omega_1,\omega_2)\omega_4,\omega_3) + S(\omega_4, W_1^*(\omega_1,\omega_2)\omega_3) \\ &= L_4\{S((\omega_1 \wedge_g \omega_2)\omega_4,\omega_3) + S(\omega_4,(\omega_1 \wedge_g \omega_2)\omega_3)\}, \end{aligned} \tag{29}$$

for $\omega_3 = \xi$ in (29), we observe

$$\begin{aligned} &S(W_1^*(\omega_1,\omega_2)\omega_4,\xi) + S(\omega_4, W_1^*(\omega_1,\omega_2)\xi) \\ &= L_4\{S(g(\omega_2,\omega_4)\omega_1 - g(\omega_1,\omega_4)\omega_2,\xi) \\ &+ S(\omega_4,\eta(\omega_2)\omega_1 - \eta(\omega_1)\omega_2)\}. \end{aligned} \tag{30}$$

Substituting (7) into (30), we obtain

$$\begin{aligned} &L_4\{-\lambda g(\omega_4,\eta(\omega_1)\omega_2 - \eta(\omega_2)\omega_1) + S(\omega_4,\eta(\omega_2)\omega_1 - \eta(\omega_1)\omega_2)\} \\ &= -\lambda\eta(W_1^*(\omega_1,\omega_2)\omega_4) + \mu S(\omega_4,\eta(\omega_2)h\omega_1 - \eta(\omega_1)h\omega_2) \\ &+ \nu S(\omega_4,\eta(\omega_2)\phi h\omega_1 - \eta(\omega_1)\phi h\omega_2). \end{aligned}$$

Also, taking into account (6) and (28), we can infer

$$\begin{aligned} &-\mu g(\eta(\omega_2)h\omega_1 - \eta(\omega_1)h\omega_2,\phi h\omega_4) + \lambda\mu\ g(\eta(\omega_1)h\omega_2 - \eta(\omega_2)h\omega_1,\omega_4) \\ &-\nu g(\phi h\omega_4,\eta(\omega_2)\phi h\omega_1 - \eta(\omega_1)\phi h\omega_2) - \lambda\nu g(\omega_4,\eta(\omega_2)\phi h\omega_1 - \eta(\omega_1)\phi h\omega_2) \\ &= -\lambda L_4 g(\eta(\omega_1)\omega_2 - \eta(\omega_2)\omega_1,\omega_4) + \lambda L_4 g(\eta(\omega_1)\omega_2 \\ &-\eta(\omega_2)\omega_1,\omega_4) - L_4 g(\eta(\omega_1)\omega_2 - \eta(\omega_2)\omega_1,\phi h\omega_4). \end{aligned}$$

Since, the ambient manifold admits an almost Ricci soliton, by using (4) and setting $\omega_1 = \xi$, we obtain

$$-\mu(1+\kappa)g(\omega_2, \phi\omega_4) + \nu(1+\kappa)g(\phi\omega_4, \phi\omega_2) + L_4 g(\phi\omega_4, h\omega_2) = 0,$$

this implies that

$$\mu(1+\kappa) = 0 \text{ and } (\nu^2 + L_4^2)(1+\kappa) = 0.$$

Thus, this completes of the proof. □

**Corollary 4.** *Let $\Xi^{2n+1}$ is a $W_1^*$-Ricci semisymmetric ${}^{(\kappa,\mu,\nu)}\mathcal{P}$-$\mathcal{CMM}$ and $(g, \xi, \lambda)$ be a Ricci soliton on $\Xi^{2n+1}$. Then, we have*

$$v^2 (1+\kappa) = 0 \text{ and } \mu (1+\kappa) = 0.$$

**Remark 4.** *If the manifold chosen in the above example is $W_1^*-$Ricci pseudosymmetric, then we have*

$$L_4 = 0 \text{ and } 5z^3 \left(\frac{5}{2}z^3 + 1\right)^2 = 0.$$

## 4. Conclusion

In this paper, we have studied Ricci solitons on $(\kappa, \mu, \nu)$-paracontact manifolds by means of the Riemann curvature tensor and the $W_0, W_1^*$ and $W_7$-curvature tensors. Our main objective was to analyze how the presence of a Ricci soliton influences curvature-restricted conditions such as Ricci pseudosymmetry and Ricci semisymmetry within the framework of paracontact geometry.

We derived several necessary conditions under which a $(\kappa, \mu, \nu)$-paracontact manifold admitting a Ricci soliton satisfies Ricci pseudosymmetric or Ricci semisymmetric properties with respect to the considered curvature tensors. These results provide new characterizations that clarify the geometric behavior of such manifolds and reveal strong interconnections between Ricci solitons and curvature restrictions. In particular, our findings show that the interaction between the soliton structure and the underlying paracontact geometry imposes significant constraints on the curvature tensors, leading to rigidity-type results in certain cases.

The obtained conclusions generalize and extend a number of earlier results in the literature concerning curvature-restricted paracontact manifolds and Ricci solitons. Moreover, they demonstrate that Ricci solitons play a crucial role in shaping both the intrinsic curvature properties and the global geometric structure of $(\kappa, \mu, \nu)$-paracontact manifolds. We expect that the techniques and results presented here will be useful for further investigations of geometric flows and curvature conditions on more general classes of paracontact and pseudo-Riemannian manifolds.

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