

Medical SuperHyperStructure and Healthcare SuperHyperStructure

Abstract

A *Hyperstructure* builds upon the notion of a powerset, offering a framework for modeling relationships among elements of a set [1, 2]. Extending this idea, a *Superhyperstructure* applies the n -th powerset to represent multi-layered hierarchical systems, allowing for deeper abstraction and higher levels of structural complexity [3, 4]. In this paper, we explore Hyperstructures and Superhyperstructures within the domains of medicine and healthcare. The proposed approach aims to provide a clearer representation of hierarchical structures inherent in these fields. It should be noted that the present study is purely theoretical. We anticipate that future work will involve quantitative analyses and experimental studies to further develop and validate these concepts.

Keywords: HyperStructure, SuperHyperStructure, Medical SuperHyperStructure, Healthcare SuperHyper-Structure

1 Preliminaries

This section gathers the core notions and notation used throughout the paper. Unless explicitly stated otherwise, all underlying sets are *finite*. By convention, we also regard the empty set \emptyset as an element of every set.

1.1 Hyperstructure and Superhyperstructure

A *Hyperstructure* arises from the powerset construction and provides a general framework for modeling relations among elements of a set [1, 2]. Thanks to its flexibility, the hyperstructure paradigm has been applied widely, including in mathematics and chemistry [5–7]. Extending this idea, a *Superhyperstructure* employs the n -th iterated powerset to encode multi-layered hierarchical relationships, enabling deeper abstraction and greater structural complexity [3, 8]. Because of its broad applicability, the superhyperstructure viewpoint has likewise been studied in mathematics, chemistry, and related areas [9, 10]. For reference, we record below the basic set-theoretic constructions on which these frameworks rest.

Definition 1.1 (Base Set). A *base set* S is the underlying collection of elements from which more elaborate constructions—such as powersets and hyperstructures—are formed. Formally,

$$S = \{x \mid x \text{ belongs to a specified domain}\}.$$

All elements of constructions like $\mathcal{P}(S)$ and the iterated powersets $\mathcal{P}_n(S)$ ultimately originate from the elements of S .

Definition 1.2 (Powerset). The *powerset* of a set S , denoted $\mathcal{P}(S)$, is the family of all subsets of S , including both \emptyset and S itself:

$$\mathcal{P}(S) = \{A \mid A \subseteq S\}.$$

Definition 1.3 (n -th Powerset). (cf. [8, 11]) For a set H , the n -th powerset $\mathcal{P}_n(H)$ is defined recursively by

$$\mathcal{P}_1(H) = \mathcal{P}(H), \quad \mathcal{P}_{n+1}(H) = \mathcal{P}(\mathcal{P}_n(H)) \quad (n \geq 1).$$

The *nonempty* n -th powerset, denoted $\mathcal{P}_n^*(H)$, is obtained analogously by removing the empty set at each stage:

$$\mathcal{P}_1^*(H) = \mathcal{P}^*(H), \quad \mathcal{P}_{n+1}^*(H) = \mathcal{P}^*(\mathcal{P}_n^*(H)),$$

where $\mathcal{P}^*(H) = \mathcal{P}(H) \setminus \{\emptyset\}$.

Example 1.4 (3rd Powerset: Weekly meal plans from dish choices). Let the base set of dishes be

$$H = \{\text{Salad, Soup, Pasta, Curry}\}.$$

Level 1 ($\mathcal{P}(H)$): **Menus**. Typical menus (subsets of dishes) are

$$M_1 = \{\text{Salad, Pasta}\}, \quad M_2 = \{\text{Soup, Curry}\}, \quad M_3 = \{\text{Salad, Soup, Curry}\} \in \mathcal{P}(H).$$

Level 2 ($\mathcal{P}^2(H) = \mathcal{P}(\mathcal{P}(H))$): **Day plan options**. A *day plan* is a set of admissible menus, e.g.

$$D_{\text{Mon}} = \{M_1, M_2\}, \quad D_{\text{Tue}} = \{M_2, M_3\} \in \mathcal{P}^2(H).$$

Level 3 ($\mathcal{P}^3(H) = \mathcal{P}(\mathcal{P}^2(H))$): **Weekly plan families**. A *weekly plan family* collects possible day plans, e.g.

$$W_\alpha = \{D_{\text{Mon}}, D_{\text{Tue}}\}, \quad W_\beta = \{D_{\text{Tue}}\} \in \mathcal{P}^3(H).$$

Thus elements of $\mathcal{P}^3(H)$ (like W_α, W_β) are sets of day-plan *sets* of menus, providing a three-tier representation:

$$\text{dishes} \rightarrow \text{menus} \rightarrow \text{day plans} \rightarrow \text{weekly plan families}.$$

If one wishes to forbid empties at each tier, replace \mathcal{P} by $\mathcal{P}^*(\cdot) = \mathcal{P}(\cdot) \setminus \{\emptyset\}$ throughout to obtain $\mathcal{P}^{*3}(H)$.

To establish a comprehensive framework for understanding Hyperstructures and Superhyperstructures, we present the following formal definitions and foundational concepts.

Definition 1.5 (Classical Structure). (cf. [8, 11, 12]) A *Classical Structure* is a mathematical framework defined on a non-empty set H , characterized by one or more *Classical Operations* that adhere to specific *Classical Axioms*. Formally:

A *Classical Operation* is a function of the form:

$$\#_0 : H^m \rightarrow H,$$

where $m \geq 1$ denotes a positive integer, and H^m represents the m -fold Cartesian product of H . Examples include algebraic operations such as addition and multiplication in structures like groups, rings, and fields.

Definition 1.6 (Hyperoperation). (cf. [3, 13, 14]) A *hyperoperation* is a generalization of a binary operation in which the result of combining two inputs is a *set* (not necessarily a singleton). Formally, for a set S , a hyperoperation \circ is a map

$$\circ : S \times S \longrightarrow \mathcal{P}(S).$$

Definition 1.7 (Hyperstructure). (cf. [8, 11, 15, 16]) A *Hyperstructure* extends the concept of a Classical Structure by operating on the powerset of a base set. It is formally defined as:

$$\mathcal{H} = (\mathcal{P}(S), \circ),$$

where S is the base set, $\mathcal{P}(S)$ denotes its powerset, and \circ is an operation defined for subsets within $\mathcal{P}(S)$.

Example 1.8 (Everyday HyperStructure: Air-travel hub suggestion). Let S be a finite set of airports

$$S = \{\text{HND}, \text{ICN}, \text{SIN}, \text{DOH}, \text{LAX}, \text{JFK}\}.$$

Assume the directed flight network (toy data)

$$\begin{aligned} \text{HND} &\rightarrow \{\text{ICN}, \text{SIN}, \text{DOH}\}, & \text{ICN} &\rightarrow \{\text{LAX}, \text{JFK}, \text{HND}\}, & \text{SIN} &\rightarrow \{\text{DOH}, \text{LAX}\}, \\ \text{DOH} &\rightarrow \{\text{LAX}, \text{JFK}, \text{HND}\}, & \text{LAX} &\rightarrow \{\text{ICN}, \text{DOH}, \text{HND}\}, & \text{JFK} &\rightarrow \{\text{HND}, \text{DOH}\}. \end{aligned}$$

Define a hyperoperation $\circ : S \times S \rightarrow \mathcal{P}(S)$ by

$$a \circ b := \{h \in S \mid a \rightarrow h \text{ and } h \rightarrow b \text{ are direct flights}\},$$

i.e. $a \circ b$ is the set of feasible *hubs* for a two-hop trip $a \rightarrow h \rightarrow b$. Then $\mathcal{H} = (\mathcal{P}(S), \circ)$ is a hyperstructure. Concrete evaluations:

$$\text{HND} \circ \text{LAX} = \{\text{ICN}, \text{SIN}, \text{DOH}\}, \quad \text{HND} \circ \text{JFK} = \{\text{ICN}, \text{DOH}\}.$$

Definition 1.9 (SuperHyperOperation). [8] Let H be a nonempty set. Define recursively, for each integer $k \geq 0$,

$$\mathcal{P}^0(H) = H, \quad \mathcal{P}^{k+1}(H) = \mathcal{P}(\mathcal{P}^k(H)).$$

Fix $m, n \geq 0$. An (m, n) -SuperHyperOperation is a map

$$\odot^{(m,n)} : (\mathcal{P}^m(H))^s \longrightarrow \mathcal{P}^n(H)$$

for some input arity $s \in \mathbb{Z}_{>0}$. When the codomain is allowed to contain \emptyset , we speak of the *Neutrosophic* variant; otherwise it is the classical variant.

Definition 1.10 (n -Superhyperstructure). (cf. [8, 11]) An n -Superhyperstructure generalizes the Hyperstructure by employing the n -th powerset of a base set. Formally, it is defined as:

$$\mathcal{SH}_n = (\mathcal{P}_n(S), \circ),$$

where S is the base set, $\mathcal{P}_n(S)$ represents the n -th powerset of S , and \circ is an operation acting on elements of $\mathcal{P}_n(S)$.

Example 1.11 (Everyday n -Superhyperstructure ($n = 2$): Round-trip hub families). Let S be the airport set above. Elements of $\mathcal{P}^2(S)$ are *families of hub-sets*. For $A, B \in \mathcal{P}^2(S)$, define

$$A \star^{[2]} B := \{ \{h_{\text{out}}, h_{\text{ret}}\} \subseteq S \mid \exists H_{\text{out}} \in A, \exists H_{\text{ret}} \in B : h_{\text{out}} \in H_{\text{out}}, h_{\text{ret}} \in H_{\text{ret}} \}.$$

Intuitively, A lists admissible *outbound* hubs and B admissible *return* hubs; the output is the family of round-trip hub pairs. Take, for the toy network,

$$A = \{ \{\text{ICN}, \text{SIN}, \text{DOH}\} \} \quad (\text{outbound HND} \rightarrow \text{LAX}), \quad B = \{ \{\text{ICN}, \text{DOH}\} \} \quad (\text{return LAX} \rightarrow \text{HND}),$$

so

$$A \star^{[2]} B = \{ \{\text{ICN}, \text{ICN}\}, \{\text{ICN}, \text{DOH}\}, \{\text{SIN}, \text{ICN}\}, \{\text{SIN}, \text{DOH}\}, \{\text{DOH}, \text{ICN}\}, \{\text{DOH}, \text{DOH}\} \} \in \mathcal{P}^2(S).$$

Hence $\mathcal{SH}_2 = (\mathcal{P}^2(S), \star^{[2]})$ is an n -superhyperstructure capturing families of round-trip hub choices.

Definition 1.12 (SuperHyperStructure of order (m, n)). (cf. [3, 10, 17, 18]) Let S be a nonempty set, and let $m, n \geq 0$. An (m, n) -SuperHyperStructure of arity s is defined by selecting a mapping

$$\odot^{(m,n)} : (\mathcal{P}^m(S))^s \longrightarrow \mathcal{P}^n(S).$$

Special cases include:

- $m = n = 0$: ordinary s -ary operations on S ,
- $m = 0, n = 1$: hyperoperations,
- $s = 1$: superhyperoperations,

Example 1.13 (Everyday (m, n) -SuperHyperStructure: Group itinerary synthesis $(m, n) = (1, 2)$). Let S be the airport set above. Define

$$\odot^{(1,2)} : (\mathcal{P}(S))^2 \longrightarrow \mathcal{P}^2(S)$$

by, for $X, Y \subseteq S$ (nonempty),

$$\odot^{(1,2)}(X, Y) := \left\{ \{h\} \subseteq S \mid \exists a \in X, \exists b \in Y : h \in a \circ b \right\}.$$

Thus $\odot^{(1,2)}$ maps *sets of origins/destinations* to a *family of single-hub options*. With $X = \{\text{HND}\}$ and $Y = \{\text{LAX}, \text{JFK}\}$ we obtain, using the computed hubs,

$$\odot^{(1,2)}(X, Y) = \{\{\text{ICN}\}, \{\text{SIN}\}, \{\text{DOH}\}\} \cup \{\{\text{ICN}\}, \{\text{DOH}\}\} = \{\{\text{ICN}\}, \{\text{SIN}\}, \{\text{DOH}\}\} \in \mathcal{P}^2(S).$$

Hence $(S, \odot^{(1,2)})$ is a concrete $(m, n) = (1, 2)$ -SuperHyperStructure: inputs live in $\mathcal{P}^1(S)$ (sets of endpoints) and outputs in $\mathcal{P}^2(S)$ (families of hub-sets).

2 Main Results

As the principal outcome of this paper, we investigate four new concepts constructed using the frameworks of HyperStructure and SuperHyperStructure.

2.1 Medical HyperStructure

Medical HyperStructure models patient states and interventions with hyperoperations mapping inputs to feasible outcome sets under guidelines, constraints, valuations.

Definition 2.1 (Medical HyperStructure). Let S_{med} be a nonempty set of *clinically distinguishable patient states*. Let Σ_{med} be a signature of *clinical actions* (diagnostic/therapeutic/triage) with arity map $\text{ar} : \Sigma_{\text{med}} \rightarrow \mathbb{Z}_{\geq 1}$. Let $(V, +, \leq)$ be a commutative, partially ordered monoid encoding clinical quantities (e.g. risk, cost, dose, utility), and let

$$\nu : S_{\text{med}} \longrightarrow V$$

be a *clinical valuation*. Let $\text{Obs} : S_{\text{med}} \rightarrow \mathcal{O}$ be an *observation map* (biomarkers, vitals, labs). For each $\sigma \in \Sigma_{\text{med}}$ fix

- a *feasibility domain* $D_{\sigma} \subseteq S_{\text{med}}^{\text{ar}(\sigma)}$,
- a *guideline correspondence* $\text{Feas}_{\sigma} : S_{\text{med}}^{\text{ar}(\sigma)} \rightarrow \mathcal{P}(S_{\text{med}})$,
- a closed convex cone $C_{\sigma} \subseteq V$ describing admissible net changes (budget/safety).

A *Medical HyperStructure* is a tuple

$$\mathbf{MHS} = (S_{\text{med}}, \Sigma_{\text{med}}, \{\sigma^S\}_{\sigma \in \Sigma_{\text{med}}}, \nu, \text{Obs}, \{\text{Feas}_{\sigma}\}, \{C_{\sigma}\}, \{D_{\sigma}\}),$$

where each *interaction hyperoperation*

$$\sigma^S : S_{\text{med}}^{\text{ar}(\sigma)} \longrightarrow \mathcal{P}(S_{\text{med}})$$

satisfies, for all $\vec{s} = (s_1, \dots, s_{\text{ar}(\sigma)})$:

(MH1) **Feasible nonemptiness:** if $\vec{s} \in D_{\sigma}$ then $\sigma^S(\vec{s}) \neq \emptyset$.

(MH2) **Guideline compatibility:** $\sigma^S(\vec{s}) \subseteq \text{Feas}_\sigma(\vec{s})$.

(MH3) **Clinical admissibility (budget/safety):** for every $u \in \sigma^S(\vec{s})$,

$$v(u) \in v(s_1) + \dots + v(s_{\text{ar}(\sigma)}) + C_\sigma.$$

(Taking $C_\sigma = \{0\}$ enforces exact conservation; $C_\sigma \neq \{0\}$ allows bounded change.)

(MH4) **Observational coherence:** there exists an aggregator $H_\sigma : \mathcal{O}^{\text{ar}(\sigma)} \rightarrow \mathcal{P}(\mathcal{O})$ such that

$$\text{Obs}(\sigma^S(\vec{s})) \subseteq H_\sigma(\text{Obs}(s_1), \dots, \text{Obs}(s_{\text{ar}(\sigma)})),$$

where $\text{Obs}(A) := \{\text{Obs}(a) \mid a \in A\}$ for $A \subseteq S_{\text{med}}$.

When one forgets $v, \text{Obs}, \text{Feas}_\sigma, C_\sigma, D_\sigma$, the pair $(S_{\text{med}}, \{\sigma^S\})$ is a plain hyperstructure in the usual sense.

Example 2.2 (Empiric antibiotics for community-acquired pneumonia (CAP)). (cf. [19]) Let S_{med} be the set of patient states $s = (\text{dx}, A, \text{sev}, r)$ consisting of diagnosis label dx , allergy set $A \subseteq \{\beta\text{-lactam}, \text{macrolide}, \dots\}$, severity class $\text{sev} \in \{1, 2, 3\}$, and current regimen r (possibly $r = \emptyset$). Let F be a hospital formulary and for $r \in F$ write $\text{cost}(r) \geq 0$ and $\text{risk}_{\text{AE}}(r) \geq 0$ (e.g. a calibrated adverse-event score).

Set the valuation space $V = \mathbb{R}_{\geq 0}^2$ with componentwise addition and order, and define

$$v(s) := (\text{cost}(r), \text{risk}_{\text{AE}}(r)).$$

Let Σ_{med} contain a unary interaction $\sigma = \text{abx_emp}$ (empiric antibiotics). Define the feasibility domain

$$D_\sigma := \{s \in S_{\text{med}} \mid \text{dx} = \text{CAP}\},$$

and the policy correspondence $\text{Feas}_\sigma(s)$ as the set of regimens $r' \in F$ that (i) cover guideline pathogens for sev , and (ii) avoid allergies in A . Fix $C_\sigma = [0, C_{\text{max}}] \times [0, R_{\text{max}}]$ (bounding incremental cost and AE risk).

Define the hyperoperation $\sigma^S : S_{\text{med}} \rightarrow \mathcal{P}(S_{\text{med}})$ by

$$\sigma^S(s) := \left\{ (\text{dx}, A, \text{sev}, r') \mid r' \in \text{Feas}_\sigma(s) \right\}.$$

Then (MH1) holds by guideline nonemptiness, (MH2) by construction, and for any $u \in \sigma^S(s)$,

$$v(u) - v(s) \in C_\sigma,$$

which is (MH3). For observational coherence (MH4), if $\text{Obs}(s)$ returns (e.g.) initial vitals/labs and microbiology (possibly pending), set $H_\sigma(o)$ to contain updated summaries after empiric therapy initiation, and observe $\text{Obs}(\sigma^S(s)) \subseteq H_\sigma(\text{Obs}(s))$.

Example 2.3 (Inpatient insulin titration for hyperglycemia). (cf. [20, 21]) Let S_{med} consist of $s = (G, d, c)$ with current capillary glucose $G \in [0, \infty)$, basal insulin dose $d \in [0, \infty)$, and clinical context c (e.g. steroid use, NPO status). Let $V = \mathbb{R}_{\geq 0}^2$ with

$$v(s) := (\text{hypo_risk}(s), \text{workload}(s)),$$

where hypo_risk is a surrogate risk (lower is better but we store it as a nonnegative load), and workload encodes nursing intensity.

Let $\sigma = \text{titrate}$ be unary. The feasibility domain is

$$D_\sigma = \{(G, d, c) \mid \text{no active contraindication to insulin adjustment in } c\}.$$

Fix step bounds $\Delta^-, \Delta^+ \geq 0$, and define the policy correspondence

$$\text{Feas}_\sigma(G, d, c) := \{d' \in [\max(0, d - \Delta^-), d + \Delta^+] \mid \text{unit protocol for } c\}.$$

Let $C_\sigma = [0, R_{\text{max}}] \times [0, W_{\text{max}}]$. Define

$$\sigma^S(G, d, c) := \left\{ (G', d', c) \mid d' \in \text{Feas}_\sigma(G, d, c), G' \in \text{Pred}(G, d, d', c) \right\},$$

where Pred is a (set-valued) next-glucose predictor consistent with the protocol (e.g. bracketing due to meal timing). Then (MH1)–(MH2) are immediate. For (MH3), protocol design yields $v(G', d', c) - v(G, d, c) \in C_\sigma$ (bounded changes in risk/workload). For (MH4), taking $\text{Obs}(s) = (G, c)$ and $H_\sigma(G, c)$ the set of admissible next measurements for protocol steps ensures $\text{Obs}(\sigma^S(s)) \subseteq H_\sigma(\text{Obs}(s))$.

Example 2.4 (Neuroimaging triage for acute head injury). (cf. [22]) Let S_{med} be states $s = (\text{GCS}, \text{on_OAC}, \text{eGFR}, \text{imaging})$ with Glasgow Coma Scale $\text{GCS} \in \{3, \dots, 15\}$, oral anticoagulant flag $\text{on_OAC} \in \{0, 1\}$, renal function $\text{eGFR} \in [0, \infty)$, and current imaging plan $\text{imaging} \in \{\emptyset, \text{CT_NC}, \text{CT_C}, \text{MRI}\}$. Let $V = \mathbb{R}_{\geq 0}^2$ with

$$\nu(s) := (\text{time_to_result}(s), \text{radiation_dose}(s)).$$

Let $\sigma = \text{neuro_triage}$ (unary). The feasibility domain requires standard trauma criteria (e.g. loss of consciousness or focal deficit \Rightarrow eligible). The policy correspondence encodes contraindications (e.g. if $\text{eGFR} < 30$ then CT_C disallowed; MRI access/time limits).

Set $C_\sigma = [0, T_{\max}] \times [0, D_{\max}]$. Define

$$\sigma^S(s) := \left\{ (\text{GCS}, \text{on_OAC}, \text{eGFR}, \text{plan}) \mid \text{plan} \in \text{Feas}_\sigma(s) \right\}.$$

Then (MH1) nonemptiness follows from at least one admissible modality; (MH2) by construction; (MH3) holds since ν changes (time, dose) are bounded by C_σ under protocolized pathways; (MH4) with $\text{Obs}(s)$ the presenting exam and FAST score, and H_σ the guideline-consistent post-triage summaries, one has $\text{Obs}(\sigma^S(s)) \subseteq H_\sigma(\text{Obs}(s))$.

Example 2.5 (Anticoagulation initiation in atrial fibrillation). (cf. [23, 24]) Let S_{med} be states

$$s = (\text{CHA2DS2-VASc}, \text{HAS-BLED}, \text{eGFR}, \text{agent})$$

, with risk scores and current agent

$$\text{agent} \in \{\emptyset, \text{DOAC_low}, \text{DOAC_std}, \text{warfarin}\}$$

. Let $V = \mathbb{R}_{\geq 0}^2$ and

$$\nu(s) := (\text{stroke_risk_est}(s), \text{bleed_risk_est}(s)).$$

Let $\sigma = \text{anticoag_start}$ (unary) with feasibility domain excluding absolute contraindications (e.g. active bleed). The policy correspondence Feas_σ selects agents/doses admissible for eGFR and risk strata per guideline. Take $C_\sigma = [-S_{\max}, B_{\max}] \times [0, B_{\max}]$ (where negative first component models a *decrease* in stroke risk within allowed range, and the second bounds bleeding-risk increase). Define

$$\sigma^S(s) := \left\{ (\text{CHA2DS2-VASc}, \text{HAS-BLED}, \text{eGFR}, \text{agent}') \mid \text{agent}' \in \text{Feas}_\sigma(s) \right\}.$$

Then (MH1)–(MH2) hold by construction. For (MH3), any $u \in \sigma^S(s)$ satisfies $\nu(u) - \nu(s) \in C_\sigma$ (bounded improvement in stroke risk and bounded change in bleeding risk). For (MH4), let $\text{Obs}(s)$ contain labs (INR, creatinine) and H_σ collect the post-initiation monitoring targets; then $\text{Obs}(\sigma^S(s)) \subseteq H_\sigma(\text{Obs}(s))$.

2.2 Medical SuperHyperStructure

Medical SuperHyperStructure extends to iterated powersets, capturing cohorts, multi-stage care pathways, and policies with lifted feasibility, observational coherence, bounded resources.

Definition 2.6 (Lifts to iterated powersets). For $k \geq 0$, define $\mathcal{P}^0((S_{\text{med}})) = S_{\text{med}}$ and $\mathcal{P}^{k+1}((S_{\text{med}})) = \mathcal{P}(\mathcal{P}^k((S_{\text{med}})))$. For $Z \in \mathcal{P}^k((S_{\text{med}}))$, its set of *atoms* is defined recursively by $\text{At}(z) = \{z\}$ when $k = 0$ and $\text{At}(Z) = \bigcup_{z \in Z} \text{At}(z)$ for $k \geq 1$. The observation map lifts as $\text{Obs}^{\uparrow 0} = \text{Obs}$ and $\text{Obs}^{\uparrow(k+1)}(X) = \{\text{Obs}^{\uparrow k}(x) \mid x \in X\}$.

Definition 2.7 (Medical SuperHyperStructure of order (m, n)). Fix integers $m, n \geq 0$ and a signature Σ_{med} with arities $\text{ar}(\sigma)$. For each $\sigma \in \Sigma_{\text{med}}$ choose

$$D_\sigma^{(m)} \subseteq (\mathcal{P}^m((S_{\text{med}})))^{\text{ar}(\sigma)}, \quad \text{Feas}_\sigma^{(m,n)} : (\mathcal{P}^m((S_{\text{med}})))^{\text{ar}(\sigma)} \longrightarrow \mathcal{P}(\mathcal{P}^n((S_{\text{med}}))),$$

and a closed convex cone $C_\sigma \subseteq V$ in the valuation space $(V, +, \leq)$ with $\nu : S_{\text{med}} \rightarrow V$ as in Definition 2.1. A *Medical SuperHyperStructure of order (m, n)* is a tuple

$$\text{MSHS} = \left(S_{\text{med}}, \Sigma_{\text{med}}, \{\odot_\sigma^{(m,n)}\}_{\sigma \in \Sigma_{\text{med}}}, \nu, \text{Obs}, \{\text{Feas}_\sigma^{(m,n)}\}, \{C_\sigma\}, \{D_\sigma^{(m)}\} \right),$$

where each *superhyperoperation*

$$\odot_\sigma^{(m,n)} : (\mathcal{P}^m((S_{\text{med}})))^{\text{ar}(\sigma)} \longrightarrow \mathcal{P}^n((S_{\text{med}}))$$

satisfies, for all inputs $\vec{X} = (X_1, \dots, X_{\text{ar}(\sigma)})$:

(MSH1) **Feasible nonemptiness:** if $\vec{X} \in D_{\sigma}^{(m)}$ then $\odot_{\sigma}^{(m,n)}(\vec{X}) \neq \emptyset$.

(MSH2) **Guideline compatibility (lifted):**

$$\odot_{\sigma}^{(m,n)}(\vec{X}) \subseteq \text{Feas}_{\sigma}^{(m,n)}(\vec{X}).$$

(MSH3) **Atomic clinical admissibility:** for every $Y \in \odot_{\sigma}^{(m,n)}(\vec{X})$ and every $u \in \text{At}(Y)$ there exist $s_i \in \text{At}(X_i)$ such that

$$\nu(u) \in \nu(s_1) + \dots + \nu(s_{\text{ar}(\sigma)}) + C_{\sigma}.$$

(MSH4) **Observational coherence (lifted):** there exists

$$H_{\sigma}^{(m,n)} : (\mathcal{P}^m(\text{()}\mathcal{O}))^{\text{ar}(\sigma)} \longrightarrow \mathcal{P}^n(\text{()}\mathcal{O})$$

with

$$\text{Obs}^{\uparrow n}(\odot_{\sigma}^{(m,n)}(\vec{X})) \subseteq H_{\sigma}^{(m,n)}(\text{Obs}^{\uparrow m}(X_1), \dots, \text{Obs}^{\uparrow m}(X_{\text{ar}(\sigma)})).$$

When $(m, n) = (0, 1)$ and $\odot_{\sigma}^{(0,1)} = \sigma^S$, Definition 2.7 reduces to the Medical HyperStructure in Definition 2.1.

Example 2.8 (Sepsis order-set generation with guideline tiers: $(m, n) = (1, 2)$). Let S_{med} consist of patient states $s = (\text{dx}, \text{sev}, A, \text{labs}, r)$ with diagnosis dx , severity class sev , allergy set A , basic labs labs , and current regimen r . Let $V = \mathbb{R}_{\geq 0}^2$ with valuation $\nu(s) = (\text{cost}(r), \text{AE_risk}(r))$. Fix an arity-1 symbol $\sigma = \text{sepsis_orders}$. For $X \in \mathcal{P}^1(\text{()}\mathcal{S}_{\text{med}}) = \mathcal{P}(S_{\text{med}}) \setminus \{\emptyset\}$, define

$$\odot_{\sigma}^{(1,2)}(X) := \left\{ \mathcal{Y} \subseteq S_{\text{med}} \mid \exists \text{ finite tier set } \mathcal{T} \text{ s.t. } \mathcal{Y} = \left\{ u \in \text{Feas}_{\sigma}^{(1,2)}(X) \mid u \in \bigcup_{s \in X} \bigcup_{t \in \mathcal{T}} \sigma_t^S(s) \right\} \right\}.$$

Here each σ_t^S is a single-tier order-set rule (e.g. “severe sepsis, β -lactam allergy”), and $\text{Feas}_{\sigma}^{(1,2)}(X)$ encodes policy filters (e.g. formulary, renal dosing). *Checks.* If every $s \in X$ meets screening criteria $D_{\sigma}^{(1)}$, the union over finitely many tiers is nonempty (MSH1). By construction $\odot_{\sigma}^{(1,2)}(X) \subseteq \text{Feas}_{\sigma}^{(1,2)}(X)$ (MSH2). For any $Y \in \odot_{\sigma}^{(1,2)}(X)$ and $u \in \text{At}(Y)$, there is $s \in \text{At}(X)$ with $u \in \sigma_t^S(s)$, hence $\nu(u) \in \nu(s) + C_{\sigma}$ for a cone $C_{\sigma} = [0, C_{\max}] \times [0, R_{\max}]$ (MSH3). With $\text{Obs}(s)$ the presenting vitals/labs and $H_{\sigma}^{(1,2)}$ collecting the post-order observable summaries at level-2, we obtain $\text{Obs}^{\uparrow 2}(\odot_{\sigma}^{(1,2)}(X)) \subseteq H_{\sigma}^{(1,2)}(\text{Obs}^{\uparrow 1}(X))$ (MSH4).

Example 2.9 (Multidisciplinary oncology consensus from proposal families: $(m, n) = (2, 1)$). Let S_{med} encode single-patient next states $s = (\text{plan}, \text{tox}, \text{cost})$ with $\text{plan} \in \{\text{Surg}, \text{Chemo}, \text{RT}, \text{Combo}\}$, and let $V = \mathbb{R}_{\geq 0}^2$ with $\nu(s) = (\text{tox}, \text{cost})$. One patient is under discussion; input $X \in \mathcal{P}^2(\text{()}\mathcal{S}_{\text{med}})$ is a family of proposal sets, e.g. one set per specialty (surgical, medical, radiation). Define the arity-1 operation $\sigma = \text{tumorboard}$ by

$$\odot_{\sigma}^{(2,1)}(X) := \left\{ u \in S_{\text{med}} \mid \exists Y \in X, \exists s \in Y \text{ with } u \in \Pi(s) \text{ and } \nu(u) - \nu(s) \in C_{\sigma} \right\},$$

where Π is a reconciliation map (e.g. dose-adjusted variants, schedule alignments) and $C_{\sigma} = \{(t, c) \mid 0 \leq t \leq T_{\max}, 0 \leq c \leq C_{\max}\}$ bounds admissible changes. *Checks.* If X contains at least one feasible proposal set $Y \in D_{\sigma}^{(2)}$, then $\odot_{\sigma}^{(2,1)}(X) \neq \emptyset$ (MSH1). Guideline filters are enforced by $u \in \text{Feas}_{\sigma}^{(2,1)}(X)$ (MSH2). For $u \in \odot_{\sigma}^{(2,1)}(X)$ we have u derived from some $s \in \text{At}(Y)$ for $Y \in X$, giving $\nu(u) \in \nu(s) + C_{\sigma}$ (MSH3). Let $\text{Obs}(s)$ collect key endpoints; $H_{\sigma}^{(2,1)}$ compresses the family-level observations to a single plan summary, hence (MSH4).

Example 2.10 (Chronic disease panel selection as bundle-of-bundles: $(m, n) = (0, 2)$). Let S_{med} be patient states $s = (\text{dx}, \text{stage}, \text{comorb})$ for a chronic condition. We generate *monitoring bundles* (labs, imaging, visits) under multiple expert templates. Let $V = \mathbb{R}_{\geq 0}^2$ with $\nu(s) = (\text{burden}(s), \text{cost}(s))$ computed from the bundle attached to s . For arity-1 symbol $\sigma = \text{monitoring}$ and $s \in S_{\text{med}}$, define

$$\odot_{\sigma}^{(0,2)}(s) := \left\{ \mathcal{Y} \subseteq S_{\text{med}} \mid \exists \text{ finite template set } \mathcal{T} \text{ s.t. } \mathcal{Y} = \left\{ \tau(s) \mid \tau \in \mathcal{T} \right\} \right\}.$$

Here each τ attaches a monitoring bundle consistent with stage/comorbidity and policy. *Checks.* Nonemptiness holds if $s \in D_{\sigma}^{(0)}$ (screened) (MSH1), and $\mathcal{Y} \subseteq \text{Feas}_{\sigma}^{(0,2)}(s)$ (MSH2). For any $Y \in \odot_{\sigma}^{(0,2)}(s)$ and $u \in \text{At}(Y)$, $\nu(u) - \nu(s) \in C_{\sigma}$ with $C_{\sigma} = [0, B_{\max}] \times [0, C_{\max}]$ bounding burden/cost (MSH3). Taking $\text{Obs}(s)$ as current schedule and $H_{\sigma}^{(0,2)}$ the level-2 collection of admissible next schedules implies (MSH4).

Example 2.11 (Renal-function-robust anticoagulation dosing: $(m, n) = (1, 1)$). Consider S_{med} with states $s = (\text{eGFR}, \text{agent}, \text{dose})$. Let $V = \mathbb{R}_{\geq 0}^2$ with $v(s) = (\text{stroke_risk}(s), \text{bleed_risk}(s))$. The input $X \in \mathcal{P}^1(S_{\text{med}})$ collects admissible *renal scenarios* (e.g. eGFR realizations from measurement uncertainty or near-term trend). For the arity-1 symbol $\sigma = \text{anticoag_robust}$, set

$$\odot_{\sigma}^{(1,1)}(X) := \left\{ u \in S_{\text{med}} \mid \exists s \in X \text{ with } u \in \text{Feas}_{\sigma}^{(1,1)}(X) \cap \sigma^S(s), v(u) - v(s) \in C_{\sigma} \right\},$$

where $\sigma^S(s)$ returns dose adjustments admissible for the specific eGFR and C_{σ} bounds the net change in risks. Then (MSH1)–(MSH2) hold by feasibility/policy; (MSH3) holds with the chosen C_{σ} ; (MSH4) follows by lifting lab/INR observations through $H_{\sigma}^{(1,1)}$ from set-level inputs to set-level outputs.

2.3 Healthcare HyperStructure

Healthcare coordinates prevention, diagnosis, treatment, and rehabilitation services, balancing patient outcomes, access, safety, cost, and workforce sustainability across populations equitably (cf. [25, 25–28]). Healthcare HyperStructure models system states with hyperoperations mapping actions to sets of feasible next states under policy and KPI constraints.

Definition 2.12 (Healthcare HyperStructure). Let S_{hcs} be a nonempty set of *healthcare system states* (e.g., tuples collecting patient cohorts, staff rosters, bed/capacity occupancy, queues, inventories). Let Σ_{hcs} be a signature of *operational actions* (triage, admit, transfer, schedule, allocate, discharge, bill, etc.) with arity map $\text{ar} : \Sigma_{\text{hcs}} \rightarrow \mathbb{Z}_{\geq 1}$.

Let $(V, +, \leq)$ be a commutative, partially ordered monoid encoding multi-criteria key performance indicators (KPI) such as *cost, workload, risk, time, utility*. Let

$$v : S_{\text{hcs}} \longrightarrow V$$

be a *system valuation* (state \mapsto KPI vector). Let $\text{Obs} : S_{\text{hcs}} \rightarrow \mathcal{O}$ be an *observation map* returning observable summaries (e.g., EHR-derived widgets: census, wait times, backlog).

For each $\sigma \in \Sigma_{\text{hcs}}$ fix:

- a *feasibility domain* $D_{\sigma} \subseteq S_{\text{hcs}}^{\text{ar}(\sigma)}$ (policy, regulatory, and physical constraints),
- a *policy correspondence* $\text{Feas}_{\sigma} : S_{\text{hcs}}^{\text{ar}(\sigma)} \rightarrow \mathcal{P}(S_{\text{hcs}})$ mapping inputs to allowed next states,
- a closed convex cone $C_{\sigma} \subseteq V$ describing admissible net KPI changes (budget/safety/capacity margins).

A *Healthcare HyperStructure* is a tuple

$$\mathbf{HHS} = \left(S_{\text{hcs}}, \Sigma_{\text{hcs}}, \{\sigma^S\}_{\sigma \in \Sigma_{\text{hcs}}}, v, \text{Obs}, \{\text{Feas}_{\sigma}\}, \{C_{\sigma}\}, \{D_{\sigma}\} \right),$$

where each *interaction hyperoperation*

$$\sigma^S : S_{\text{hcs}}^{\text{ar}(\sigma)} \longrightarrow \mathcal{P}(S_{\text{hcs}})$$

satisfies, for all $\vec{s} = (s_1, \dots, s_{\text{ar}(\sigma)})$:

(HH1) **Feasible nonemptiness:** if $\vec{s} \in D_{\sigma}$ then $\sigma^S(\vec{s}) \neq \emptyset$.

(HH2) **Policy compatibility:** $\sigma^S(\vec{s}) \subseteq \text{Feas}_{\sigma}(\vec{s})$.

(HH3) **KPI admissibility (budget/safety/capacity):** for every $u \in \sigma^S(\vec{s})$,

$$v(u) \in v(s_1) + \dots + v(s_{\text{ar}(\sigma)}) + C_{\sigma}.$$

(Choosing $C_{\sigma} = \{0\}$ enforces exact KPI additivity; $C_{\sigma} \neq \{0\}$ allows bounded deviations.)

(HH4) **Observational coherence:** there exists an aggregator $H_\sigma : \mathcal{O}^{\text{ar}(\sigma)} \rightarrow \mathcal{P}(\mathcal{O})$ with

$$\text{Obs}(\sigma^S(\vec{s})) \subseteq H_\sigma(\text{Obs}(s_1), \dots, \text{Obs}(s_{\text{ar}(\sigma)})),$$

where $\text{Obs}(A) := \{\text{Obs}(a) \mid a \in A\}$ for $A \subseteq S_{\text{hcs}}$.

For binary σ the induced operation on $\mathcal{P}(S_{\text{hcs}})$ is $A \sigma B := \bigcup_{a \in A, b \in B} \sigma^S(a, b)$, as standard in hyperalgebra.

Example 2.13 (ED triage and admission policy (arity 1)). (cf. [29, 30]) Let S_{hcs} collect ED system states $s = (q, b, r)$ where $q = (q_1, \dots, q_L) \in \mathbb{N}^L$ are queue counts by acuity tier, $b \in \mathbb{N}$ is the number of free inpatient beds, and $r \in \mathbb{R}_{\geq 0}^d$ aggregates available resources (nurses, bays). Let $V = \mathbb{R}_{\geq 0}^2$ with componentwise $+$ and \leq , and set

$$\nu(s) := (\text{wait_proxy}(s), \text{overtime_proxy}(s)).$$

Let $\sigma = \text{triage_admit}$ be unary. The feasibility domain D_σ requires (e.g.) r above a minimal safe staffing threshold. Define the policy correspondence

$$\begin{aligned} \text{Feas}_\sigma(s) := \Big\{ s' = (q', b', r') \mid \exists u \in \mathbb{N}^L : \sum_{\ell=1}^L u_\ell \leq b, 0 \leq u_\ell \leq q_\ell, \\ q' = q - u, \quad b' = b - \sum_{\ell} u_\ell, \quad r' = \varphi(r, u), \text{ and all unit rules hold} \Big\}, \end{aligned}$$

where φ updates resources (e.g., assigns nurse load). Fix a cone $C_\sigma = \{(x_1, x_2) \in \mathbb{R}_{\geq 0}^2 \mid x_1 \leq W_{\max}, x_2 \leq O_{\max}\}$. Define the hyperoperation

$$\sigma^S(s) := \text{Feas}_\sigma(s).$$

Then (HH1) holds whenever $b > 0$ or a fast-track is available; (HH2) is by definition; for any $u \in \sigma^S(s)$ one has $\nu(u) - \nu(s) \in C_\sigma$ (HH3), i.e. KPI changes are bounded; and with $\text{Obs}(s)$ the live dashboard (census, queue, staffing), one can take $H_\sigma(\text{Obs}(s))$ as the set of post-admit dashboards allowed by policy (HH4).

Example 2.14 (Operating room (OR) day scheduling under block policy (arity 1)). (cf. [31, 32]) Let S_{hcs} contain states $s = (\mathcal{J}, B, \kappa)$ where \mathcal{J} is a finite set of surgical cases with predicted durations $\{d_j\}_{j \in \mathcal{J}}$, B are block assignments per service, and κ summarizes turnover times and staffing. Let

$$V = \mathbb{R}_{\geq 0}^2, \quad \nu(s) := (\text{overtime}(s), \text{idle_time}(s)).$$

Let $\sigma = \text{build_schedule}$ (unary). Feasibility D_σ enforces block ownership, precedence constraints, and room limits. Define

$$\begin{aligned} \text{Feas}_\sigma(s) := \Big\{ s' = (\mathcal{J}, B, \kappa') \mid \exists \text{feasible timetable } \tau : \text{rooms} \times \text{time} \rightarrow \mathcal{J} \cup \{\emptyset\}, \\ \text{satisfying blocks } B, \text{ turnovers under } \kappa, \text{ and capacity limits; } \kappa' = \psi(\kappa, \tau) \Big\}. \end{aligned}$$

Let $C_\sigma = \{(x_1, x_2) \in \mathbb{R}_{\geq 0}^2 \mid x_1 \leq O_{\max}, x_2 \leq I_{\max}\}$. Set

$$\sigma^S(s) := \text{Feas}_\sigma(s).$$

Then (HH1) follows from existence of a feasible packing (e.g. first-fit with overtime cap); (HH2) is immediate; (HH3) holds by the KPI bounds; and $\text{Obs}(s)$ (case list, blocks) passes through an aggregator H_σ (Gantt snapshots, KPIs), giving (HH4).

Example 2.15 (Inpatient bed management with inter-unit transfers (arity 1)). (cf. [33, 34]) Let S_{hcs} be hospital states $s = (\mathbf{c}, \mathbf{k}, \iota)$ where $\mathbf{c} = (c_1, \dots, c_U)$ are current occupancies per unit, $\mathbf{k} = (k_1, \dots, k_U)$ are capacities, and ι collects isolation/cohorting constraints (matrix of compatibilities). Let $V = \mathbb{R}_{\geq 0}^2$ with

$$\nu(s) := (\text{boarding_hours}(s), \text{transfer_load}(s)).$$

Let $\sigma = \text{transfer}$ be unary. Feasibility D_σ requires free capacity respecting isolation. Define the policy correspondence as

$$\text{Feas}_\sigma(s) := \Big\{ s' = (\mathbf{c}', \mathbf{k}, \iota) \mid \exists \text{flow } f_{uv} \in \mathbb{N}_{\geq 0} \text{ with} \Big.$$

$$\mathbf{c}' = \mathbf{c} + \sum_v f_v \cdot - \sum_u f_u \cdot, 0 \leq c'_u \leq k_u, \text{ and cohorting constraints respected} \}.$$

Let $C_\sigma = \{(x_1, x_2) \in \mathbb{R}_{\geq 0}^2 \mid x_1 \leq B_{\max}, x_2 \leq L_{\max}\}$. Set

$$\sigma^S(s) := \text{Feas}_\sigma(s).$$

Then (HH1) holds whenever a feasible integer flow exists (classical max-flow with side constraints); (HH2) is by definition; (HH3) holds since boarding and transfer workload changes are bounded; (HH4) uses $\text{Obs}(s)$ (unit census, blocks) and H_σ (bed board after transfers).

Example 2.16 (Pharmacy inventory replenishment with service-level targets (arity 1)). (cf. [35, 36]) Let S_{hcs} be pharmacy states $s = (x, \hat{d}, L)$ with current on-hand vector $x \in \mathbb{R}_{\geq 0}^n$, forecasted demand $\hat{d} \in \mathbb{R}_{\geq 0}^n$, and supplier lead times $L \in \mathbb{R}_{> 0}^n$. Let

$$V = \mathbb{R}_{\geq 0}^2, \quad v(s) := (\text{stockout_risk}(s), \text{holding_cost}(s)).$$

Let $\sigma = \text{replenish}$ (unary) with feasibility domain enforcing order windows and budget. Define

$$\text{Feas}_\sigma(s) := \left\{ s' = (x + q, \hat{d}, L) \mid q \in \mathcal{Q}(s) \subseteq \mathbb{R}_{\geq 0}^n, \mathbf{1}^\top c \odot q \leq \text{Budget}, \text{ service levels met} \right\},$$

where $\mathcal{Q}(s)$ is the admissible order set and c the unit costs. Take $C_\sigma = \{(x_1, x_2) \in \mathbb{R}_{\geq 0}^2 \mid x_1 \leq R_{\max}, x_2 \leq H_{\max}\}$. Then

$$\sigma^S(s) := \text{Feas}_\sigma(s)$$

satisfies (HH1)–(HH3) by construction; with $\text{Obs}(s)$ the inventory dashboard and H_σ the post-order KPIs (fill rate curves), (HH4) holds.

Example 2.17 (Outpatient clinic overbooking policy (arity 1)). (cf. [37, 38]) Let S_{hcs} consist of $s = (\mathcal{A}, \rho, \theta)$ where \mathcal{A} is the current appointment list, $\rho \in [0, 1]$ a no-show estimate, and θ resource parameters (rooms, providers). Let $V = \mathbb{R}_{\geq 0}^2$ with

$$v(s) := (\text{expected_wait}(s), \text{overtime_risk}(s)).$$

Let $\sigma = \text{overbook}$ be unary. Feasibility D_σ enforces policy bounds (e.g. overbooking cap per hour). Define

$$\text{Feas}_\sigma(s) := \left\{ s' = (\mathcal{A}', \rho, \theta) \mid \mathcal{A}' \in \mathcal{G}(\mathcal{A}, \rho, \theta) \text{ (admissible insertions/deferrals), service constraints satisfied} \right\}.$$

Let $C_\sigma = \{(x_1, x_2) \in \mathbb{R}_{\geq 0}^2 \mid x_1 \leq W_{\max}, x_2 \leq O_{\max}\}$. Set $\sigma^S(s) = \text{Feas}_\sigma(s)$. Then (HH1)–(HH3) hold immediately, and with $\text{Obs}(s)$ the booked template and KPI projections, the aggregator H_σ returns the set of post-policy templates and projections, giving (HH4).

2.4 Healthcare SuperHyperStructure

Healthcare SuperHyperStructure extends hyperstructures using iterated powersets, capturing hierarchical, multi-facility decisions with lifted feasibility, observation coherence, and bounded performance changes.

Definition 2.18 (Lifts, atoms, and lifted observations). For $k \geq 0$, define $\mathcal{P}^0((S_{\text{hcs}})) = S_{\text{hcs}}$ and $\mathcal{P}^{k+1}((S_{\text{hcs}})) = \mathcal{P}(\mathcal{P}^k((S_{\text{hcs}})))$. For $Z \in \mathcal{P}^k((S_{\text{hcs}}))$, its set of *atoms* is defined by $\text{At}(z) = \{z\}$ when $k = 0$ and $\text{At}(Z) = \bigcup_{z \in Z} \text{At}(z)$ for $k \geq 1$. The observation map lifts as $\text{Obs}^{\uparrow 0} = \text{Obs}$ and $\text{Obs}^{\uparrow(k+1)}(X) = \{\text{Obs}^{\uparrow k}(x) \mid x \in X\}$.

Definition 2.19 (Healthcare SuperHyperStructure of order (m, n)). Fix integers $m, n \geq 0$ and a signature Σ_{hcs} with arities $\text{ar}(\sigma)$. For each $\sigma \in \Sigma_{\text{hcs}}$ choose

$$D_\sigma^{(m)} \subseteq (\mathcal{P}^m((S_{\text{hcs}})))^{\text{ar}(\sigma)}, \quad \text{Feas}_\sigma^{(m,n)} : (\mathcal{P}^m((S_{\text{hcs}})))^{\text{ar}(\sigma)} \longrightarrow \mathcal{P}(\mathcal{P}^n((S_{\text{hcs}}))),$$

and a closed convex cone $C_\sigma \subseteq V$ in the KPI monoid $(V, +, \leq)$ together with the valuation $v : S_{\text{hcs}} \rightarrow V$ from Definition 2.12. A *Healthcare SuperHyperStructure of order (m, n)* is a tuple

$$\mathbf{HSHS} = (S_{\text{hcs}}, \Sigma_{\text{hcs}}, \{\odot_\sigma^{(m,n)}\}_{\sigma \in \Sigma_{\text{hcs}}}, v, \text{Obs}, \{\text{Feas}_\sigma^{(m,n)}\}, \{C_\sigma\}, \{D_\sigma^{(m)}\}),$$

where each *superhyperoperation*

$$\odot_\sigma^{(m,n)} : (\mathcal{P}^m((S_{\text{hcs}})))^{\text{ar}(\sigma)} \longrightarrow \mathcal{P}^n((S_{\text{hcs}}))$$

satisfies, for all inputs $\vec{X} = (X_1, \dots, X_{\text{ar}(\sigma)})$:

(HSH1) **Feasible nonemptiness (lifted)**: if $\vec{X} \in D_{\sigma}^{(m)}$ then $\odot_{\sigma}^{(m,n)}(\vec{X}) \neq \emptyset$.

(HSH2) **Policy compatibility (lifted)**:

$$\odot_{\sigma}^{(m,n)}(\vec{X}) \subseteq \text{Feas}_{\sigma}^{(m,n)}(\vec{X}).$$

(HSH3) **Atomic KPI admissibility**: for every $Y \in \odot_{\sigma}^{(m,n)}(\vec{X})$ and every $u \in \text{At}(Y)$ there exist $s_i \in \text{At}(X_i)$ with

$$\nu(u) \in \nu(s_1) + \dots + \nu(s_{\text{ar}(\sigma)}) + C_{\sigma}.$$

(HSH4) **Observational coherence (lifted)**: there exists

$$H_{\sigma}^{(m,n)} : (\mathcal{P}^m(\text{()}\mathcal{O}))^{\text{ar}(\sigma)} \longrightarrow \mathcal{P}^n(\text{()}\mathcal{O})$$

such that

$$\text{Obs}^{\uparrow n}(\odot_{\sigma}^{(m,n)}(\vec{X})) \subseteq H_{\sigma}^{(m,n)}(\text{Obs}^{\uparrow m}(X_1), \dots, \text{Obs}^{\uparrow m}(X_{\text{ar}(\sigma)})).$$

When $(m, n) = (0, 1)$ and $\odot_{\sigma}^{(0,1)} = \sigma^S$, this reduces to the Healthcare HyperStructure of Definition 2.12.

Example 2.20 (Regional ED surge activation from scenario ensembles: $(m, n) = (1, 2)$). Let S_{hcs} be ED network states $s = (q, b, r)$ with queue vector $q \in \mathbb{N}^L$, free beds $b \in \mathbb{N}$, and resources $r \in \mathbb{R}_{\geq 0}^d$. Let $V = \mathbb{R}_{\geq 0}^2$ and $\nu(s) = (\text{wait_proxy}(s), \text{overtime_proxy}(s))$. Consider a unary symbol $\sigma = \text{surge_activate}$. Inputs are nonempty sets $X \in \mathcal{P}^1(\text{()}\mathcal{S}_{\text{hcs}}) = \mathcal{P}(S_{\text{hcs}}) \setminus \{\emptyset\}$, e.g. *demand scenarios* collected over a short horizon.

Fix a finite family \mathcal{T} of *surge tiers* (tiered policies: staffing call-ins, diversions, field triage). For $t \in \mathcal{T}$ let $\sigma_t^S : S_{\text{hcs}} \rightarrow \mathcal{P}(S_{\text{hcs}})$ be the tier- t hyperoperation (deterministic or set-valued under uncertainty), and let $\text{Feas}_{\sigma}^{(1,2)}(X)$ encode systemwide regulatory/operational filters. Choose a KPI cone $C_{\sigma} = \{(x_1, x_2) \mid 0 \leq x_1 \leq W_{\max}, 0 \leq x_2 \leq O_{\max}\}$.

Define the superhyperoperation

$$\odot_{\sigma}^{(1,2)}(X) := \left\{ \mathcal{Y} \subseteq S_{\text{hcs}} \mid \exists \mathcal{T}' \subseteq \mathcal{T} \text{ finite, } \mathcal{Y} = \left\{ u \in \bigcup_{s \in X} \bigcup_{t \in \mathcal{T}'} \sigma_t^S(s) \right\} \wedge \mathcal{Y} \subseteq \text{Feas}_{\sigma}^{(1,2)}(X) \right\}.$$

Checks. If every $s \in X$ passes screening $D_{\sigma}^{(1)}$, some tier set \mathcal{T}' yields a nonempty union, hence (HSH1). Inclusion by definition gives (HSH2). For any $Y \in \odot_{\sigma}^{(1,2)}(X)$ and $u \in \text{At}(Y)$, there exist $s \in \text{At}(X)$ and $t \in \mathcal{T}'$ with $u \in \sigma_t^S(s)$, so $\nu(u) - \nu(s) \in C_{\sigma}$ (HSH3). With Obs the live dashboard, let $H_{\sigma}^{(1,2)}$ be the level-2 collection of dashboards admissible after tier activation; then $\text{Obs}^{\uparrow 2}(\odot_{\sigma}^{(1,2)}(X)) \subseteq H_{\sigma}^{(1,2)}(\text{Obs}^{\uparrow 1}(X))$ (HSH4).

Example 2.21 (Multi-site OR block-swap consensus from proposal families: $(m, n) = (2, 1)$). Let S_{hcs} contain hospital-day states $s = (\mathcal{J}, B, \kappa)$ (cases \mathcal{J} , block map B , turnover/staff κ). Let $V = \mathbb{R}_{\geq 0}^2$ with $\nu(s) = (\text{overtime}(s), \text{idle_time}(s))$. A single day is planned across sites; the input $X \in \mathcal{P}^2(\text{()}\mathcal{S}_{\text{hcs}})$ is a *family of feasible proposal sets* (one set per site/service).

Let Π be a reconciliation map that transforms a proposed timetable s into a *network-consistent* timetable $u = \Pi(s)$ (e.g. swapping blocks, synchronizing anesthesia teams), subject to policy $\text{Feas}_{\sigma}^{(2,1)}(X)$ and KPI cone $C_{\sigma} = \{(x_1, x_2) \mid 0 \leq x_1 \leq O_{\max}, 0 \leq x_2 \leq I_{\max}\}$. Define for unary $\sigma = \text{or_consensus}$:

$$\odot_{\sigma}^{(2,1)}(X) := \left\{ u \in S_{\text{hcs}} \mid \exists Y \in X, \exists s \in Y : u = \Pi(s), u \in \text{Feas}_{\sigma}^{(2,1)}(X), \nu(u) - \nu(s) \in C_{\sigma} \right\}.$$

Checks. If some $Y \in X$ is feasible ($Y \in D_{\sigma}^{(2)}$), then $\odot_{\sigma}^{(2,1)}(X) \neq \emptyset$ (HSH1). Policy is enforced by membership in $\text{Feas}_{\sigma}^{(2,1)}(X)$ (HSH2). Atomic admissibility holds since u is derived from an atom $s \in \text{At}(Y)$ with bounded KPI drift (HSH3). If Obs returns case/room snapshots and KPIs, let $H_{\sigma}^{(2,1)}$ aggregate family-level observations into a single consensus snapshot; then (HSH4) holds.

Example 2.22 (Pharmacy procurement under lead-time uncertainty: $(m, n) = (1, 1)$). (cf. [39]) Let S_{hcs} be pharmacy states $s = (x, \hat{d}, L)$ (on-hand x , demand forecast \hat{d} , lead times L). Let $V = \mathbb{R}_{\geq 0}^2$ with $\nu(s) = (\text{stockout_risk}(s), \text{holding_cost}(s))$. An input $X \in \mathcal{P}^1(\text{()}\mathcal{S}_{\text{hcs}})$ collects *lead-time scenarios* consistent with current suppliers. Let $\mathcal{Q}(X)$ be the admissible order vectors under budget/service constraints evaluated *robustly* over X . Fix $C_{\sigma} = \{(r, h) \mid 0 \leq r \leq R_{\max}, 0 \leq h \leq H_{\max}\}$.

For unary $\sigma = \text{robust_replenish}$ define

$$\odot_{\sigma}^{(1,1)}(X) := \left\{ u = (x + q, \hat{d}, L') \in S_{\text{hcs}} \mid \exists s = (x, \hat{d}, L) \in X, \exists q \in Q(X) : \right. \\ \left. u \in \text{Feas}_{\sigma}^{(1,1)}(X) \wedge v(u) - v(s) \in C_{\sigma} \right\}.$$

Checks. Nonemptiness holds when a robust order q exists (HSH1). Policy feasibility is built into $\text{Feas}_{\sigma}^{(1,1)}$ (HSH2). Taking the witness $s \in \text{At}(X)$ for u gives atomic KPI admissibility (HSH3). With Obs the inventory dashboard and $H_{\sigma}^{(1,1)}$ the set of post-order KPI projections across X , we obtain (HSH4).

Example 2.23 (Outpatient network template bank selection: $(m, n) = (0, 2)$). (cf. [40]) Let S_{hcs} contain clinic states $s = (\mathcal{A}, \rho, \theta)$ (appointments \mathcal{A} , no-show estimate ρ , resources θ). Let $V = \mathbb{R}_{\geq 0}^2$ with $v(s) = (\text{expected_wait}(s), \text{overtime_risk}(s))$. Fix a finite *template bank* \mathfrak{T} (overbooking/capacity templates per hour/clinic) and let $\tau(s) \in \mathcal{P}(S_{\text{hcs}})$ be the set of next states yielded by template $\tau \in \mathfrak{T}$. Let $C_{\sigma} = \{(w, o) \mid 0 \leq w \leq W_{\max}, 0 \leq o \leq O_{\max}\}$.

For unary $\sigma = \text{template_select}$ and $s \in S_{\text{hcs}}$, set

$$\odot_{\sigma}^{(0,2)}(s) := \left\{ \mathcal{Y} \subseteq S_{\text{hcs}} \mid \exists \mathcal{T}' \subseteq \mathfrak{T} \text{ finite s.t. } \mathcal{Y} = \{u \in \tau(s) \mid \tau \in \mathcal{T}'\} \wedge \mathcal{Y} \subseteq \text{Feas}_{\sigma}^{(0,2)}(s) \right\}.$$

Checks. Screening $s \in D_{\sigma}^{(0)}$ ensures some admissible template, hence nonemptiness (HSH1). Inclusion into $\text{Feas}_{\sigma}^{(0,2)}(s)$ gives (HSH2). For any $Y \in \odot_{\sigma}^{(0,2)}(s)$ and $u \in \text{At}(Y)$, we have $u \in \tau(s)$ for some τ and $v(u) - v(s) \in C_{\sigma}$ (HSH3). With Obs the current schedule dashboard, let $H_{\sigma}^{(0,2)}$ collect the family of post-template dashboards to obtain (HSH4).

3 Conclusion

In this paper, we explored Hyperstructures and Superhyperstructures within the domains of medicine and healthcare. Looking ahead, we hope that future research will extend these concepts by incorporating frameworks such as Fuzzy Sets [41, 42], Intuitionistic Fuzzy Sets [43, 44], Neutrosophic Sets [45, 46], HyperFuzzy Sets [47, 48], and Plithogenic Sets [49–51]. We also anticipate that quantitative analyses will be carried out to further validate and enhance these theoretical developments.

Data Availability

This paper is theoretical and did not generate or analyze any empirical data. We welcome future studies that apply and test these concepts in practical settings.

Research Integrity

The author confirms that this manuscript is original, has not been published elsewhere, and is not under consideration by any other journal.

Use of Computational Tools

All proofs and derivations were performed manually; no computational software (e.g., Mathematica, SageMath, Coq) was used.

Code Availability

No code or software was developed for this study.

Ethical Approval

This research did not involve human participants or animals, and therefore did not require ethical approval.

Use of Generative AI and AI-Assisted Tools

We use generative AI and AI-assisted tools for tasks such as English grammar checking, and We do not employ them in any way that violates ethical standards.

Supplementary Information

No supplementary materials accompany this paper.

Disclaimer

The ideas presented here are theoretical and have not yet been validated through empirical testing. While we have strived for accuracy and proper citation, inadvertent errors may remain. Readers should verify any referenced material independently. The opinions expressed are those of the authors and do not necessarily reflect the views of their institutions.

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